Overview

Modelling

Topics to be recalled and covered include:

- How to select the appropriate model complexity
- How to build models for a given plant
- How to describe model errors.
- How to linearise nonlinear models

It also provides a brief overview to certain commonly used models, including

- State space models
- High order differential and high order difference equation models

The Need for Models

The power of a mathematical model lies in the fact that it can be simulated in hypothetical situations, be subject to states that would be dangerous in reality, and it can be used as a basis for synthesizing controllers.

Model Complexity

In building a model, it is important to bear in mind that all real processes are complex and hence any attempt to build an exact description of the plant is usually an impossible goal. Fortunately, feedback is usually very forgiving and hence, in the context of control system design, one can usually get away with rather simple models, provided they capture the essential features of the problem.

Building Models

A first possible approach to building a plant model is to postulate a specific model structure and to use what is known as a *black box* approach to modeling. In this approach one varies, either by trial and error or by an algorithm, the model parameters until the dynamic behavior of model and plant match sufficiently well.

An alternative approach for dealing with the modeling problem is to use physical laws (such as conservation of mass, energy and momentum) to construct the model. In this approach one uses the fact that, in any real system, there are *basic phenomenological laws* which determine the relationships between all the signals in the system.

In practice, it is common to combine both black box and phenomenological ideas to building a model.

Control relevant models are often quite simple compared to the true process and usually combine physical reasoning with experimental data.

State Space Models

For continuous time systems

$$\frac{dx}{dt} = f(x(t), u(t), t)$$
$$y(t) = g(x(t), u(t), t)$$

For discrete time systems

$$x[k+1] = f_d(x[k], u[k], k)$$
$$y[k] = g_d(x[k], u[k], k)$$

Linear State Space Models

$$\frac{dx(t)}{dt} = \mathbf{A}x(t) + \mathbf{B}u(t)$$
$$y(t) = \mathbf{C}x(t) + \mathbf{D}u(t)$$

Summary

- In order to systematically design a controller for a particular system, one needs a formal - though possibly simple - description of the system. Such a description is called a model.
- A model is a set of mathematical equations that are intended to capture the effect of certain system variables on certain other system variables.

- The italicized expressions above should be understood as follows:
  - *Certain system variables*: It is usually neither possible nor necessary to model the effect of every variable on every other variable; one therefore limits oneself to certain subsets. Typical examples include the effect of input on output, the effect of disturbances on output, the effect of a reference signal change on the control signal, or the effect of various unmeasured internal system variables on each other.

- *Capture*: A model is never perfect and it is therefore always associated with a modeling error. The word capture highlights the existence of errors, but does not yet concern itself with the precise definition of their type and effect.
- *Intended*: This word is a reminder that one does not always succeed in finding a model with the desired accuracy and hence some iterative refinement may be needed.
- *Set of mathematical equations*: There are numerous ways of describing the system behavior, such as linear or nonlinear differential or difference equations.

- Models are classified according to properties of the equation they are based on. Examples of classification include:

Model Attribute	*Contrasting Attribute*	*Asserts whether or not …*
Single input Single output	Multiple input multiple output	… the model equations have one input and one output only
Linear	Nonlinear	… the model equations are linear in the system variables
Time varying	Time invariant	… the model parameters are constant
Continuous	Sampled	… model equations describe the behavior at every instant of time, or only in discrete *samples* of time
Input-output	State space	… the model equations rely on functions of input and output variables only, or also include the so called *state variables*.
Lumped parameter	Distributed parameter	… the model equations are ordinary or partial differential equations