Overview

Classical PID Control
(continuous-time)

This lecture recalls and examines a particular control structure that has become almost universally used in industrial control. It is based on a particular fixed structure controller family, the so-called PID controller family. These controllers have proven to be robust and extremely beneficial in the control of many important applications.

PID stands for:

P (*Proportional*)

I (*Integral*)

D (*Derivative*)

PID Structure

Consider the simple SISO control loop shown in Figure 6.1:

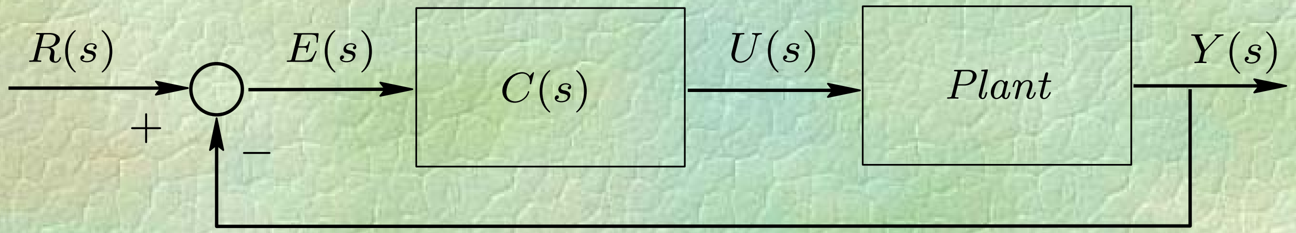


Figure 6.1: *Basic feedback control loop*

The *standard form* PID are:

Proportional only: $C_P(s) = K_p$

Proportional plus Integral: $C_{PI}(s) = K_p\left(1 + \dfrac{1}{T_r s}\right)$

Proportional plus derivative: $C_{PD}(s) = K_p\left(1 + \dfrac{T_d s}{\tau_D s + 1}\right)$

Proportional, integral and derivative: $C_{PID}(s) = K_p\left(1 + \dfrac{1}{T_r s} + \dfrac{T_d s}{\tau_D s + 1}\right)$

Tuning of PID Controllers

Because of their widespread use in practice, we present below several methods for tuning PID controllers. *Actually these methods are quite old and date back to the 1950's*. Nonetheless, they remain in widespread use today.

In particular, we will consider the:

- *Ziegler-Nichols Oscillation Method*

Ziegler-Nichols (Z-N) Oscillation Method

This procedure is only valid for open loop stable plants and it is carried out through the following steps

- Set the true plant under proportional control, with a very small gain.
- Increase the gain until the loop starts oscillating. Note that linear oscillation is required and that it should be detected at the controller output.

- Record the controller critical gain $K_p = K_c$ and the oscillation period of the controller output, P_c.
- Adjust the controller parameters according to Table 6.1 (*next slide*); there is some controversy regarding the PID parameterization for which the Z-N method was developed, but the version described here is, to the best knowledge of the authors, applicable to the parameterization of standard form PID.

Table 6.1: *Ziegler-Nichols tuning using the oscillation method*

	$\mathbf{K_p}$	$\mathbf{T_r}$	$\mathbf{T_d}$
P	$0.50K_c$		
PI	$0.45K_c$	$\dfrac{P_c}{1.2}$	
PID	$0.60K_c$	$0.5P_c$	$\dfrac{P_c}{8}$

Numerical Example

Consider a plant with a model given by

$$G_o(s) = \frac{1}{(s+1)^3}$$

Find the parameters of a PID controller using the Z-N oscillation method. Obtain a graph of the response to a unit step input reference and to a unit step input disturbance.

Solution

Applying the procedure we find:

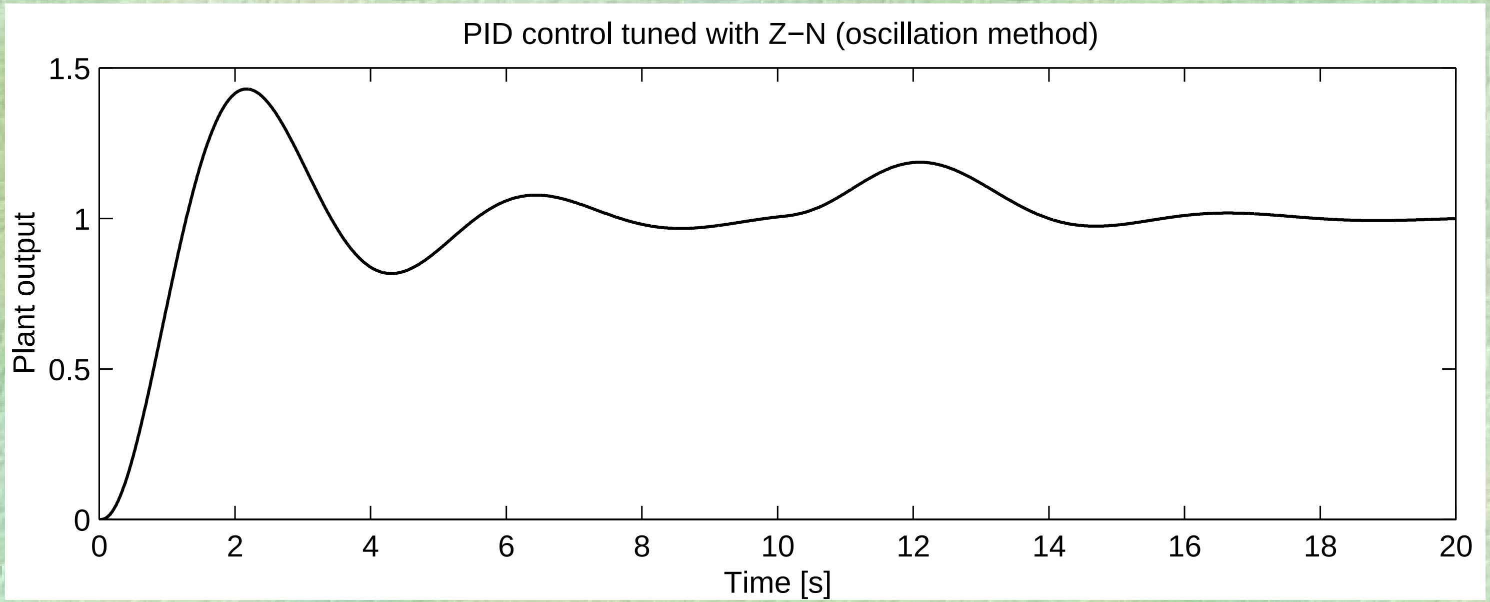
$$K_c = 8 \text{ and } \omega_c = \sqrt{3}.$$

Hence, from Table 6.1, we have

$$K_p = 0.6 * K_c = 4.8; \qquad T_r = 0.5 * P_c \approx 1.81; \qquad T_d = 0.125 * P_c \approx 0.45$$

The closed loop response to a unit step in the reference at $t = 0$ and a unit step disturbance at $t = 10$ are shown in the next figure.

Figure 6.4: *Response to step reference and step input disturbance*



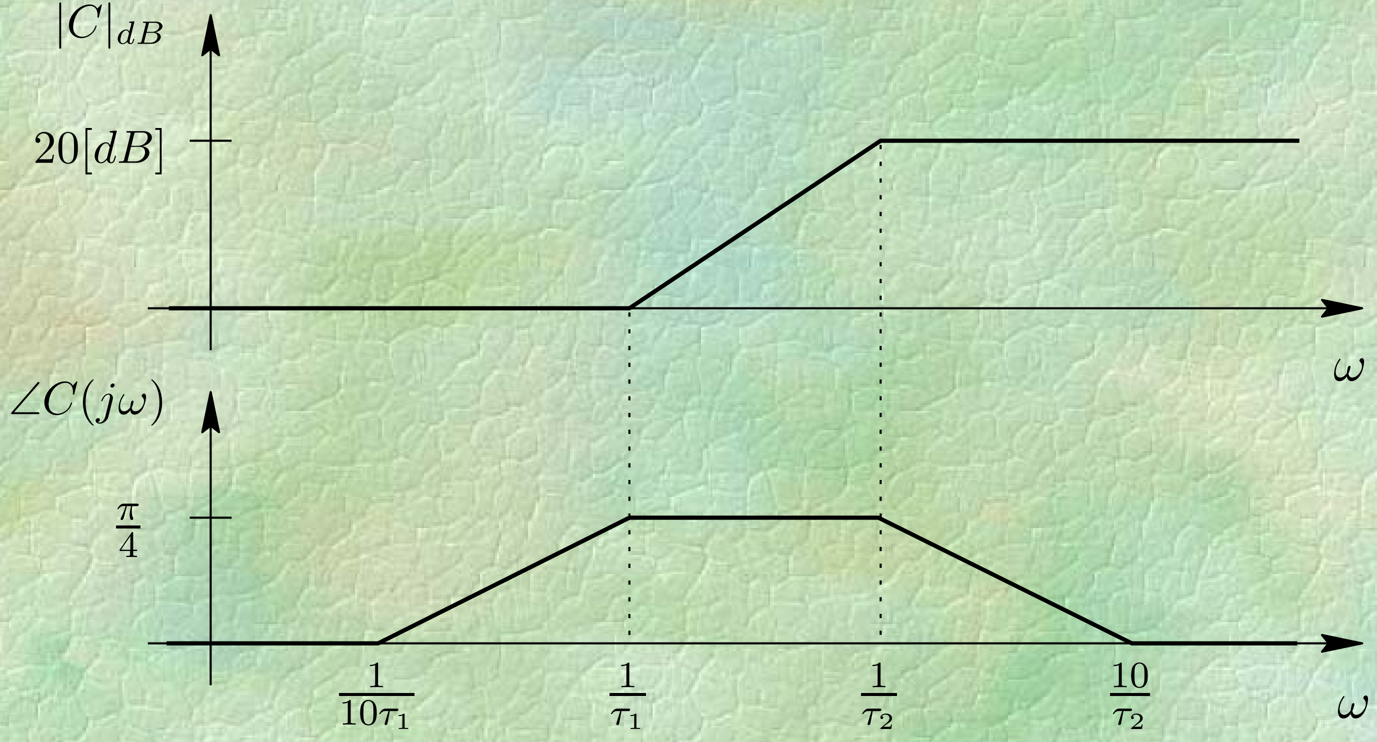
Lead-lag Compensators

Closely related to PID control is the idea of lead-lag compensation. The transfer function of these compensators is of the form:

$$C(s) = \frac{\tau_1 s + 1}{\tau_2 s + 1}$$

If $\tau_1 > \tau_2$, then this is a *lead network* and when $\tau_1 < \tau_2$, this is a *lag network*.

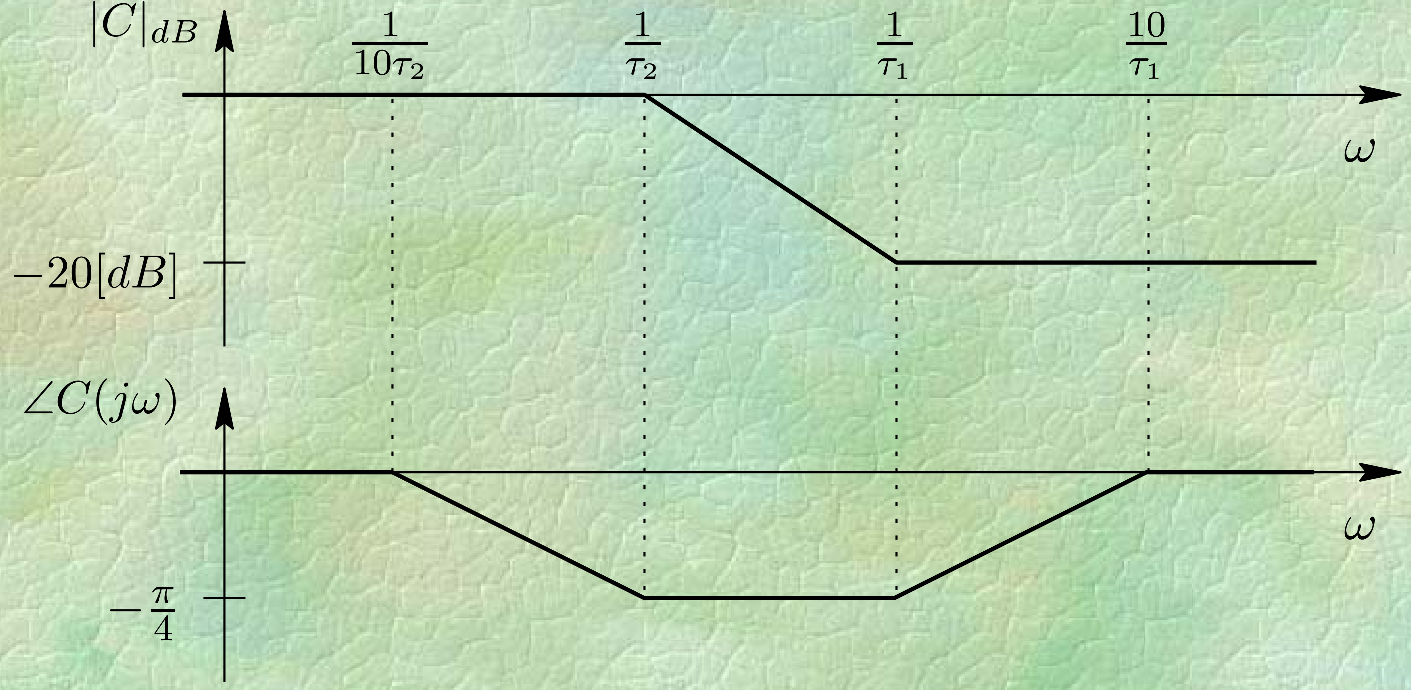
Figure 6.9: *Approximate Bode diagrams for **lead networks** ($\tau_1=10\tau_2$)*



Observation

We see from the previous slide that the lead network gives phase advance at $\omega = 1/\tau_1$ without an increase in gain. Thus it plays a role similar to derivative action in PID.

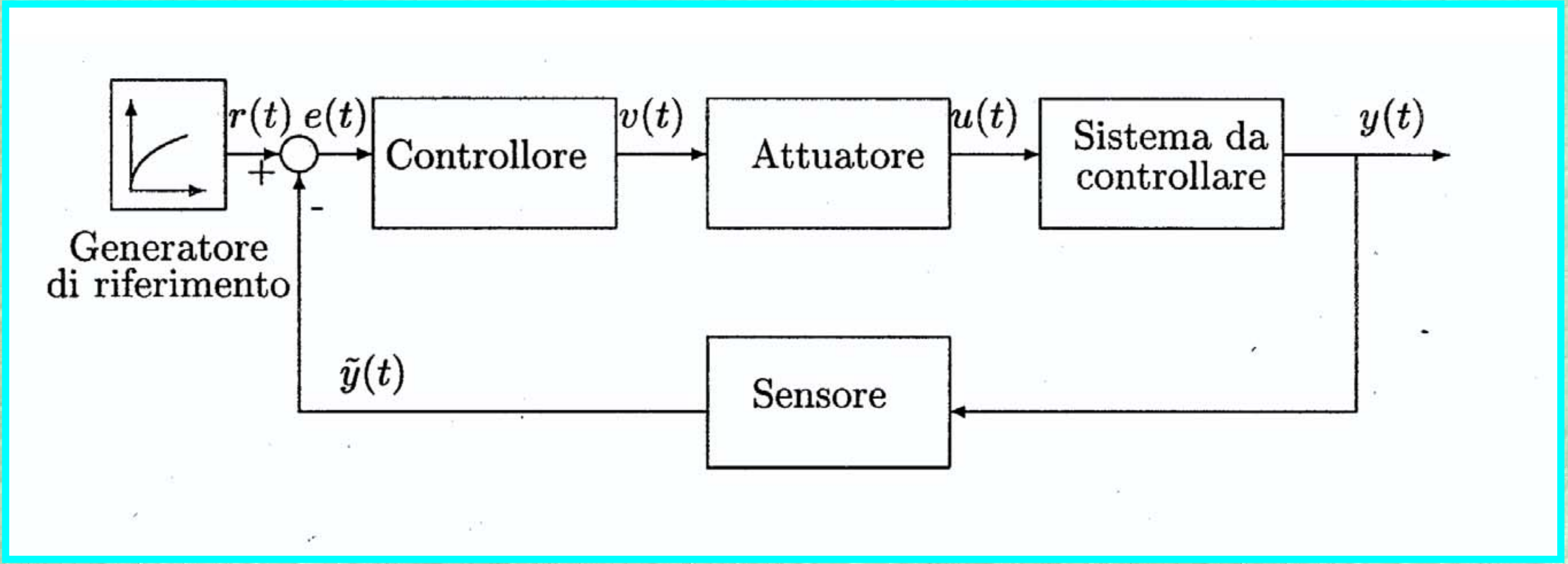
Figure 6.10: *Approximate Bode diagrams for* ***lag networks*** *(τ_2=10τ_1)*



Observation

We see from the previous slide that the lag network gives low frequency gain increase. Thus it plays a role similar to integral action in PID.

Industrial Controllers



General controlled system structure

PID Functional Blocks

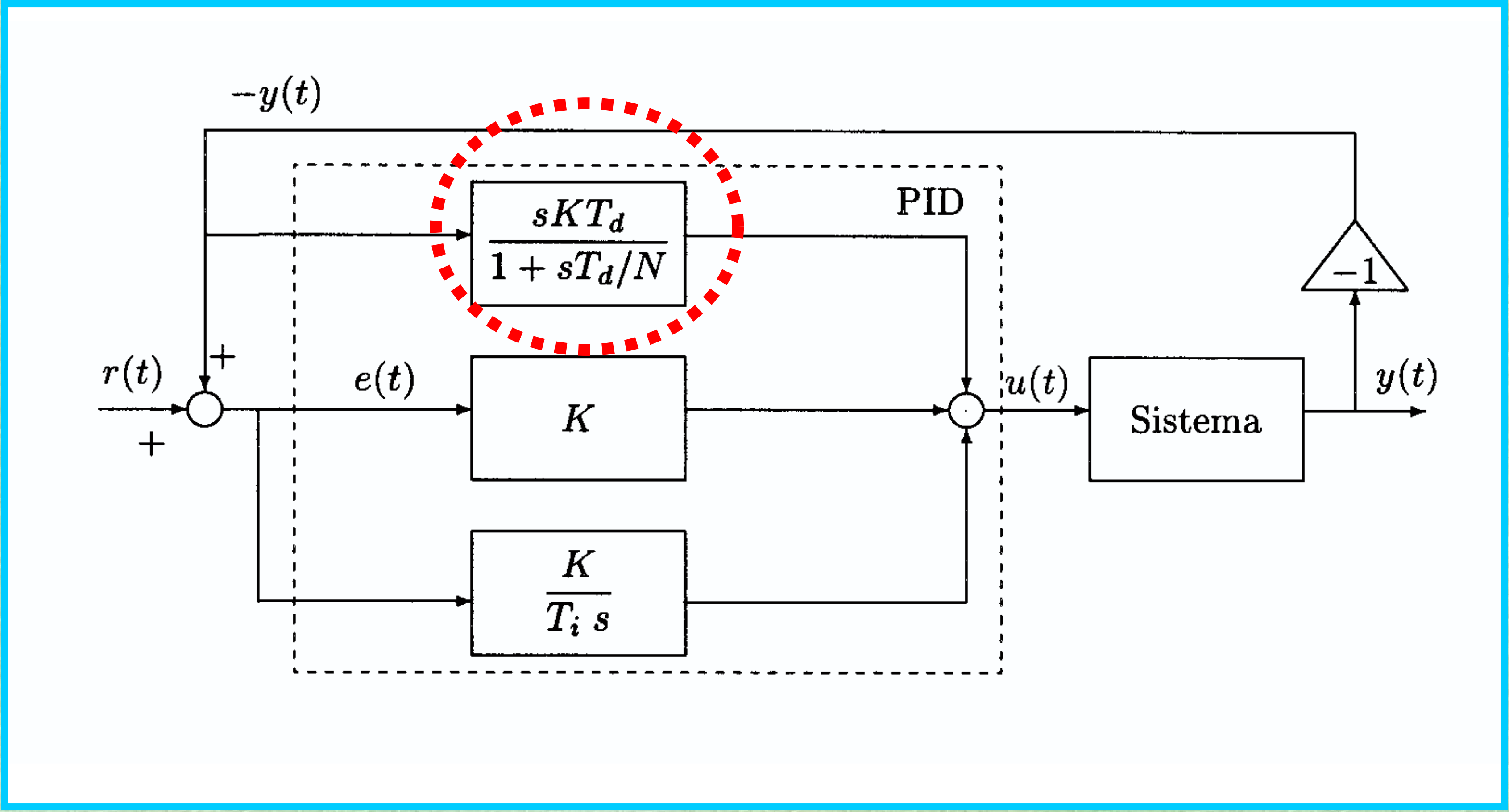
$$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\right)$$

But... Unfeasible controller!!!

$$D(S) = \frac{sKT_d}{1 + sT_d/N}$$

Feasible derivative contribution

"Real" PID Controller



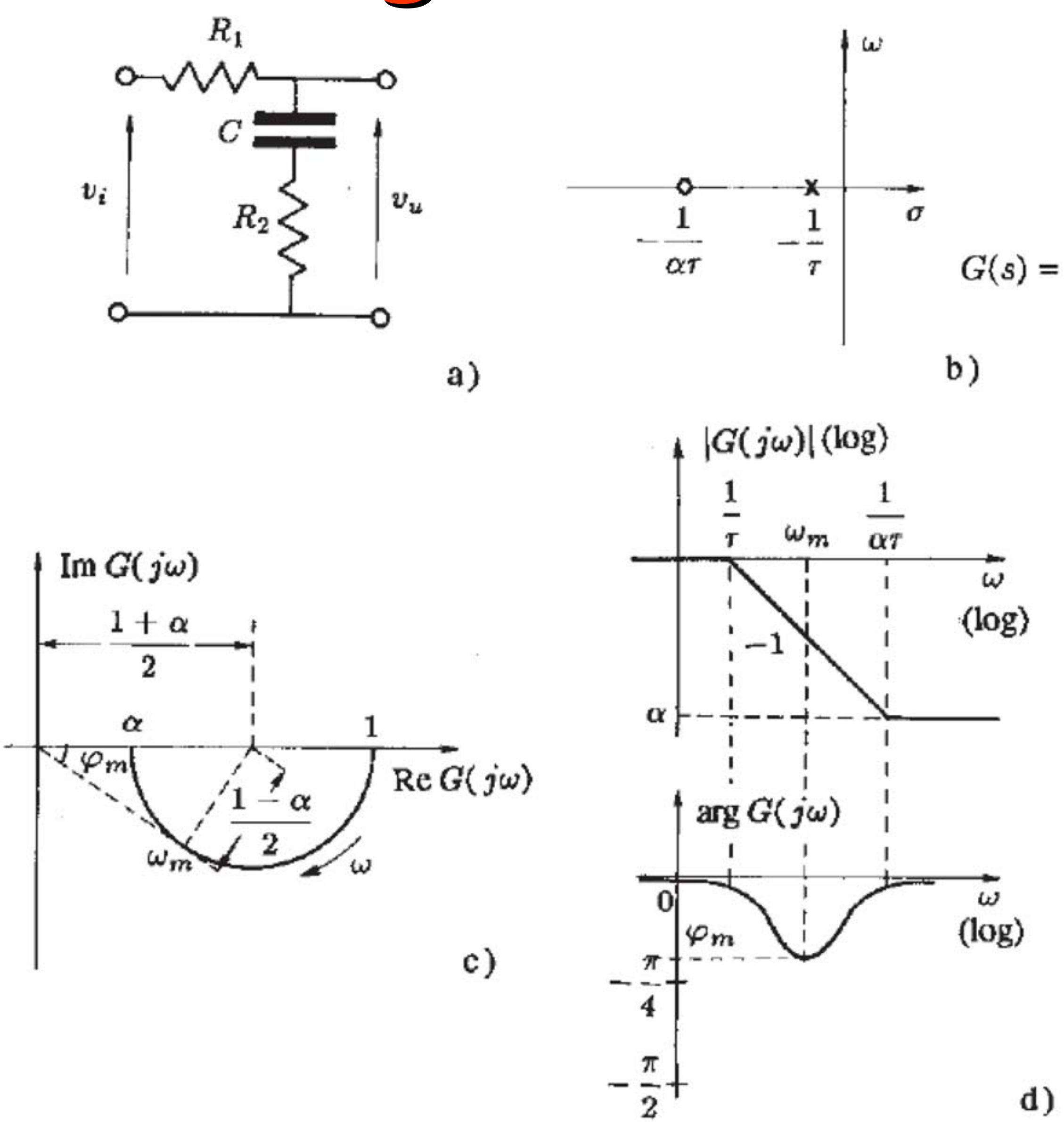
Continuous Time Controller Designs

Tools:

Bode Diagrams

Nichols Charts

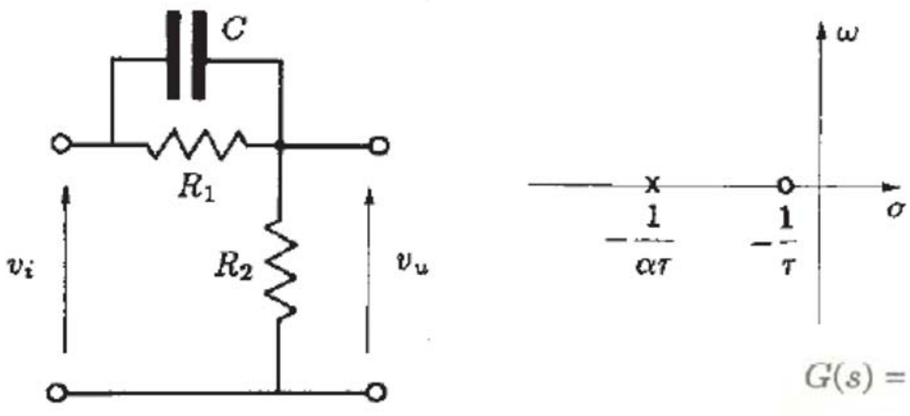
Lag Network Example



$$G(s)=\frac{R_2+\frac{1}{C\,s}}{R_1+R_2+\frac{1}{C\,s}}=\frac{1+R_2\,C\,s}{1+(R_1+R_2)\,C\,s}=\frac{1+\alpha\,\tau\,s}{1+\tau\,s}$$

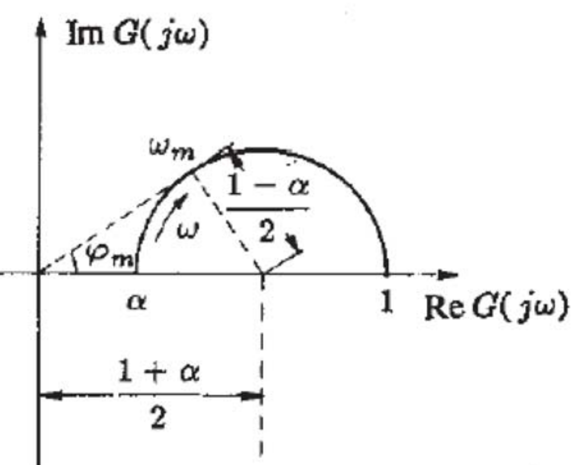
$$\varphi_m=-\text{arcsen}\,\frac{1-\alpha}{1+\alpha}\,,\qquad \omega_m=\frac{1}{\tau\sqrt{\alpha}}\,.$$

Lead Network Example

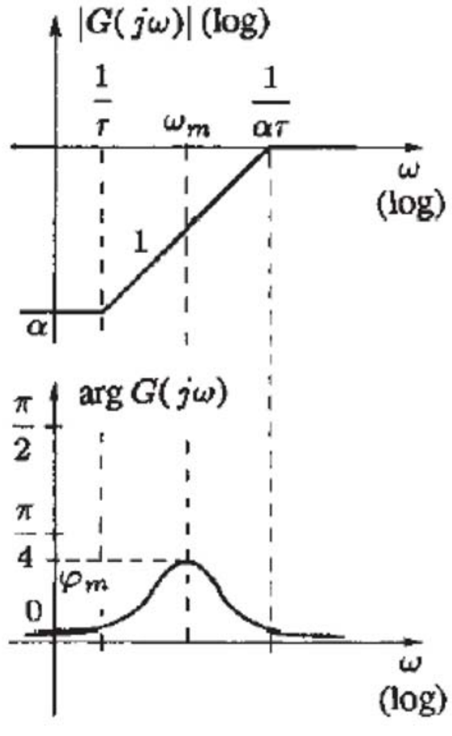


a)

$$G(s) = \frac{R_2}{R_2 + \frac{1}{(1/R_1) + C\,s}} = \frac{R_2\,(1 + R_1\,C\,s)}{R_1 + R_2 + R_1\,R_2\,C\,s} = \alpha\,\frac{1 + \tau\,s}{1 + \alpha\,\tau\,s},$$



c)

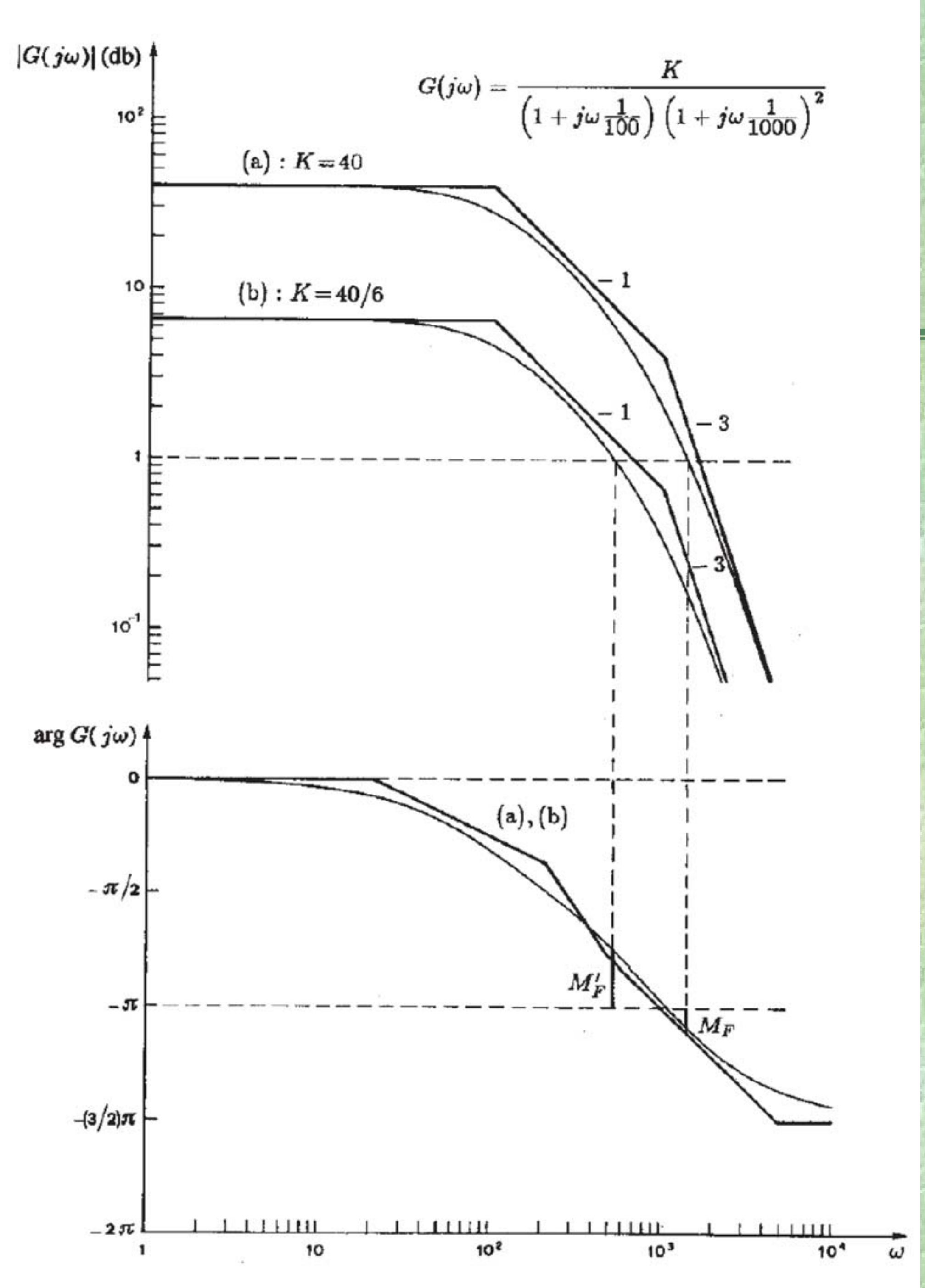


d)

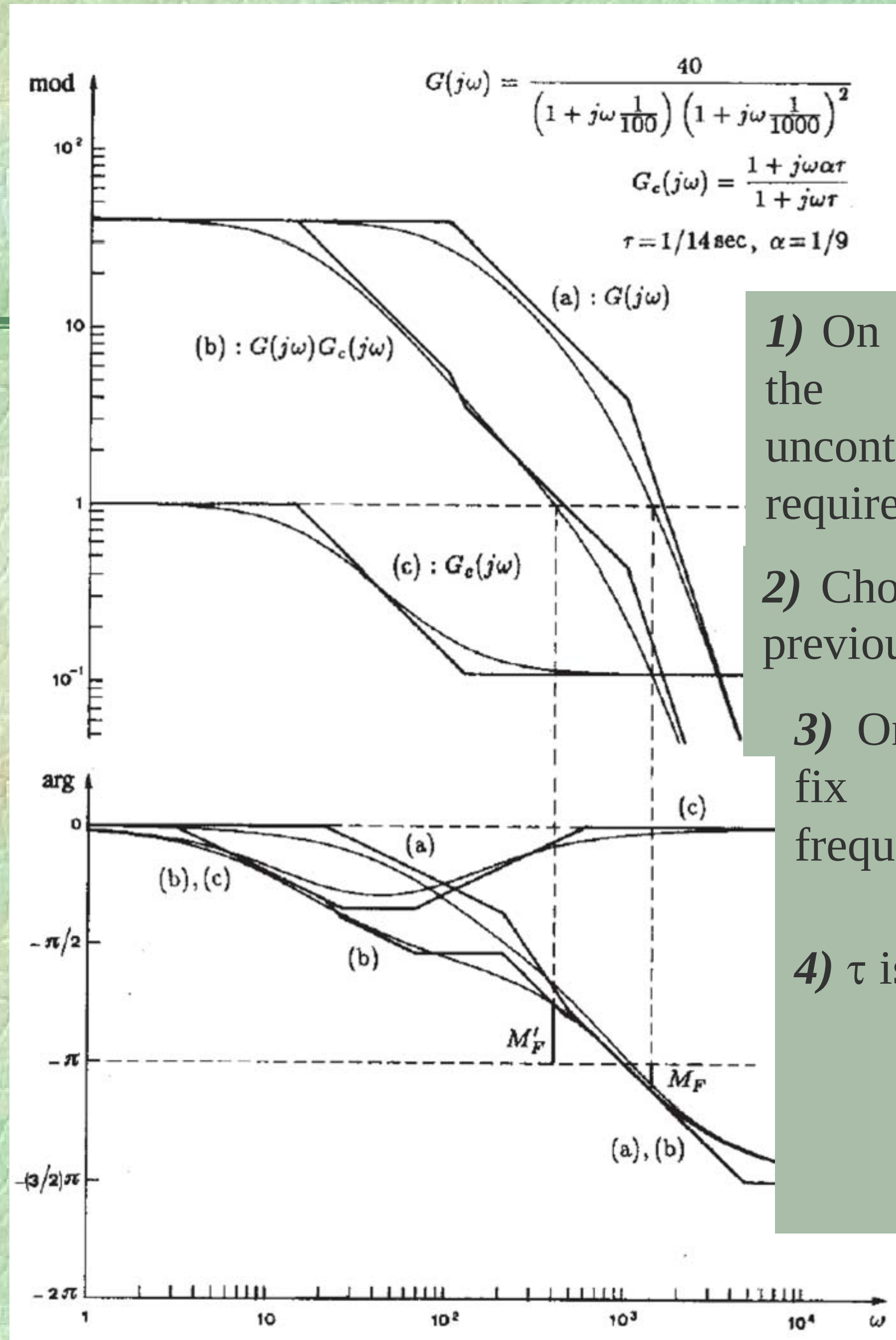
$$\omega_m = \frac{1}{\tau\sqrt{\alpha}}.$$

$$\varphi_m = \text{arcsen}\,\frac{1-\alpha}{1+\alpha}.$$

Proportional Controller



Lag network controller



1) On the Bode phase diagram fix the frequency where the uncontrolled system has the required M_f.

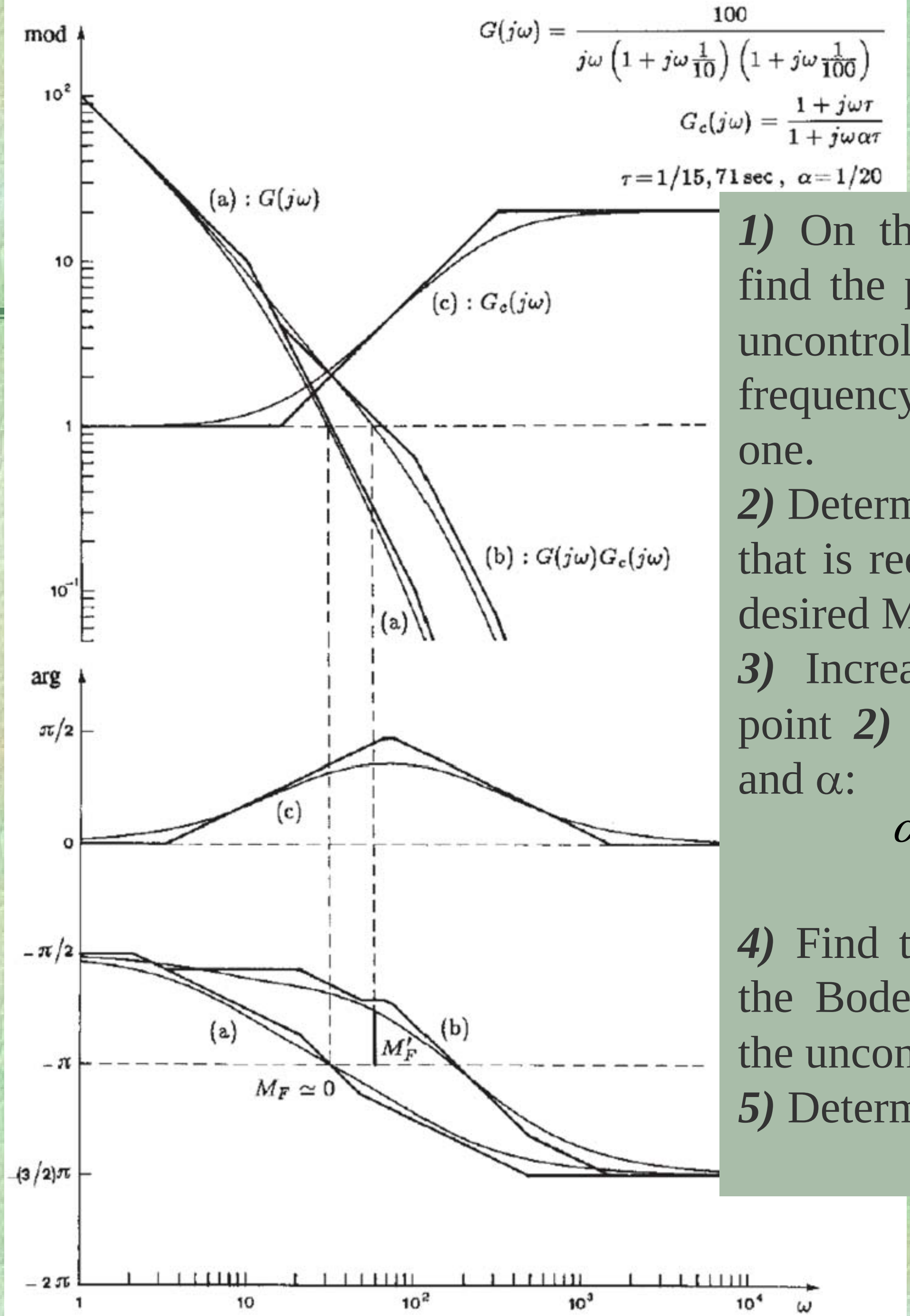
2) Choose ω^* 20% less than the previous fixed frequency.

3) On the Bode magnitude plot fix the attenuation at the frequency ω^* (i.e. the value of α).

4) τ is computed by the relation:

$$\frac{1}{\alpha\,\tau} = \frac{\omega^*}{10}$$

Lead network controller



1) On the Bode phase diagram find the phase margin M_f of the uncontrolled system and the ω* frequency where its magnitude is one.

2) Determine the basic phase lead that is required for achieving the desired M_f .

3) Increase the value fixed at point **2)** and thus determine ϕ_m and α:

$$\alpha = \frac{1 - \sin(\phi_m)}{1 + \sin(\phi_m)}$$

4) Find the frequency ω^* where the Bode magnitude diagram of the uncontrolled model is α /2.

5) Determine τ from the relation:

$$\omega^* = \frac{1}{\tau\sqrt{\alpha}}$$

Summary

- PI and PID controllers are widely used in industrial control.
- From a modern perspective, a PID controller is simply a controller of (up to second order) containing an integrator. Historically, however, PID controllers were tuned in terms of their **P**, **I** and **D** terms.
- It has been empirically found that the PID structure often has sufficient flexibility to yield excellent results in many applications.

- The basic term is the proportional term, **P**, which causes a corrective control actuation proportional to the error.

- The integral term, **I** gives a correction proportional to the integral of the error. This has the positive feature of ultimately ensuring that sufficient control effort is applied to reduce the tracking error to zero. However, integral action tends to have a destabilizing effect due to the increased phase shift.

- The derivative term, **D**, gives a predictive capability yielding a control action proportional to the rate of change of the error. This tends to have a stabilizing effect but often leads to large control movements.
- Various empirical tuning methods can be used to determine the PID parameters for a given application. They should be considered as a first guess in a search procedure.