Diagnosi Automatica Dei Guasti

"FAULT DIAGNOSIS OF DYNAMIC SYSTEMS USING MODEL-BASED AND FILTERING APPROACHES"

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January 2009

Annual Meetings on FDI

- IFAC SAFEPROCESS Symposium
- *Symposium on Fault Detection Supervision and Safety for Technical Processes*
  - 1st held in Baden–Baden, Germany in 1991
  - 2nd in Espo, Finland in 1994
  - 3rd at Hull, UK in 1997
  - 4th held in Budapest, Hungary in 2000
  - 5th at Washington DC in July 2003
  - 6th in Beijing, P.R. China, August 2006
  - 7th scheduled in Barcelona, Spain in 2009

Nomenclature

1. *States and Signals*

Fault
An unpermitted deviation of at least one characteristic property or parameter of the system from the acceptable, usual or standard condition.

Failure
A permanent interruption of a system's ability to perform a required function under specified operating conditions.

Malfunction
An intermittent irregularity in the fulfilment of a system's desired function.

Error
A deviation between a measured or computed value of an output variable and its true or theoretically correct one.

Nomenclature (cont'd)

1. *States and Signals*

Disturbance
An unknown and uncontrolled input acting on a system.

Residual
A fault indicator, based on a deviation between measurements and model-equation-based computations.

Symptom
A change of an observable quantity from normal behaviour.

Nomenclature (Cont'd)

2. *Functions*

Fault detection
Determination of faults present in a system and the time of detection.

Fault isolation
Determination of the kind, location and time of detection of a fault. Follows fault detection.

Fault identification
Determination of the size and time-variant behaviour of a fault. Follows fault isolation.

Fault diagnosis
Determination of the kind, size, location and time of detection of a fault. Follows fault detection. Includes fault detection and identification.

Monitoring
A continuous real-time task of determining the conditions of a physical system, by recording information, recognising and indication anomalies in the behaviour.

Supervision
Monitoring a physical and taking appropriate actions to maintain the operation in the case of fault.

Nomenclature (Cont'd)

3. *Models*

Quantitative model
Use of static and dynamic relations among system variables and parameters in order to describe a system's behaviour in quantitative mathematical terms.

Qualitative model
Use of static and dynamic relations among system variables in order to describe a system's behaviour in qualitative terms such as causalities and IF–THEN rules.

Diagnostic model
A set of static or dynamic relations which link specific input variables, *the symptoms*, to specific output variables, the faults.

Analytical redundancy
Use of more (not necessarily identical) ways to determine a variable, where one way uses a mathematical process model in analytical form.

Nomenclature (Cont'd)

Reliability
Ability of a system to perform a required function under stated conditions, within a given scope, during a given period of time.

Safety
Ability of a system not to cause danger to persons or equipment or the environment.

Availability
Probability that a system or equipment will operate satisfactorily and effectively at any point of time.

Nomenclature (Cont'd)

5. *Time dependency of faults*

Abrupt fault
Fault modelled as stepwise function. It represents bias in the monitored signal.

Incipient fault
Fault modelled by using ramp signals. It represents drift of the monitored signal.

Intermittent fault
Combination of impulses with different amplitudes.

NOTE: Incipient fault = hard to detect !!!

Nomenclature (Cont'd)

6. *Fault terminology*

Additive fault

Influences a variable by an addition of the fault itself. They may represent, *e.g.*, offsets of sensors.

Multiplicative fault

Are represented by the product of a variable with the fault itself. They can appear as parameter changes within a process.

Application Examples

- Simulated case studies
- Identification/FDI applications
- Real processes
- Research works
- Undergraduate theses topics

Simulated Application Examples

Simulated Gas Turbine

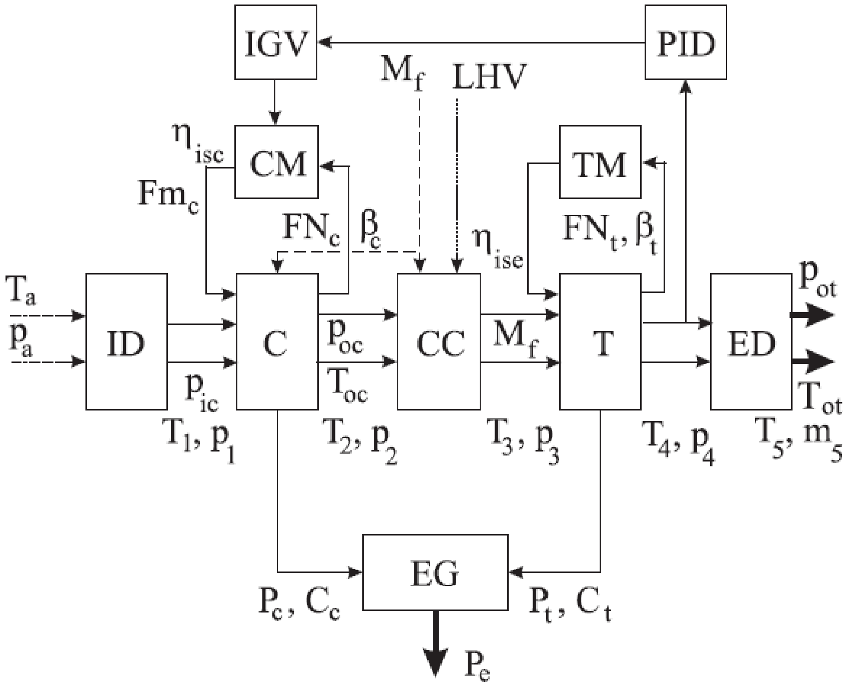


Figure 5.2: Block diagram of the single–shaft gas turbine.

C	Compressor
CC	Combustor (Combustion Chamber)
CM	Compressor Map
ED	Exhaust Duct
EG	Electric Generator
ID	Intake Duct
IGV	Inlet Guide Vanes
PID	Proportional Integral Derivative Controller
T	Turbine
TM	Turbine Map

Simulated Application Examples (Cont'd)

Simulated Gas Turbine

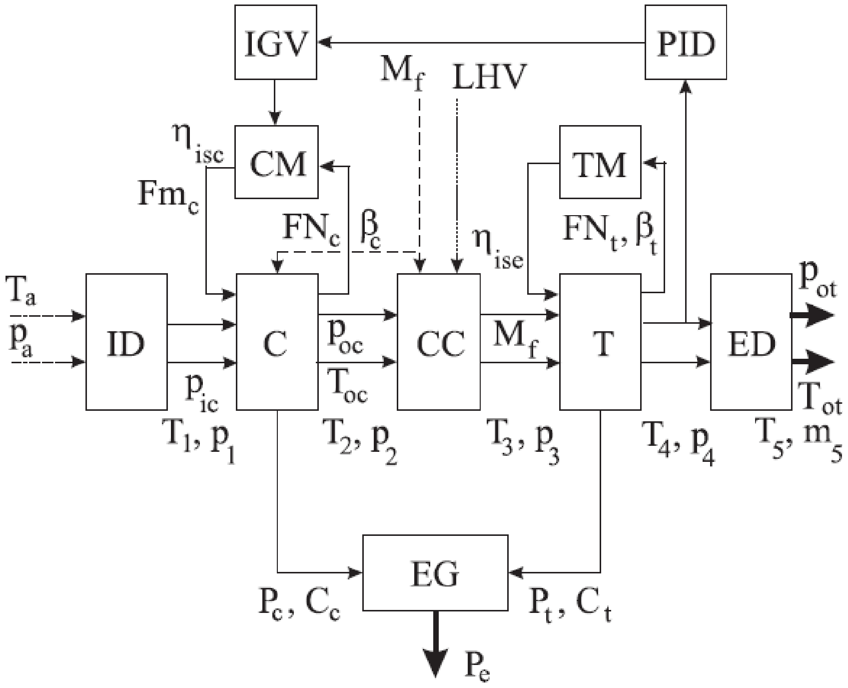
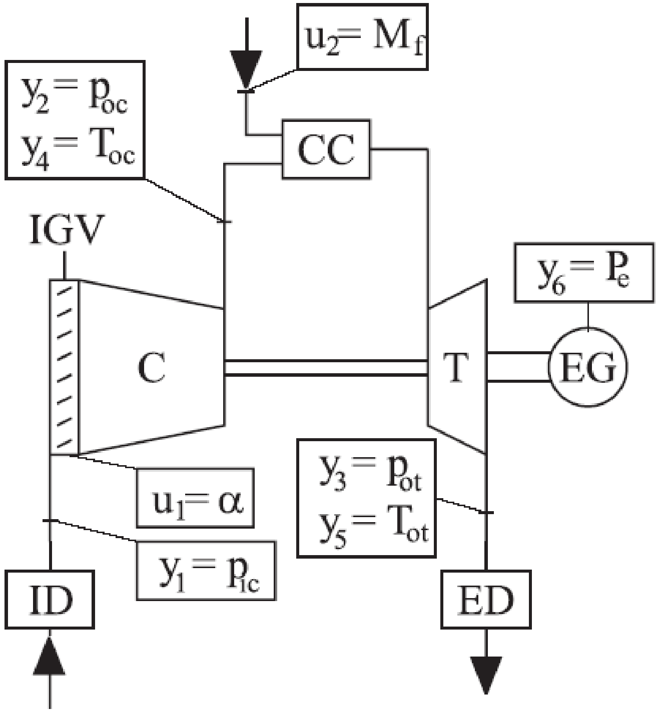


Figure 5.2: Block diagram of the single–shaft gas turbine.

M_f	Fuel mass flow rate
LHV	Lower Heating Value
η_{isc}	Isentropic compressor efficiency
Fm_c	Compressor mass flow function
FN_c	Compressor rotational speed function
β_c	Compressor pressure ratio
η_{ise}	Isentropic expansion efficiency
FN_t	Turbine rotational speed function
β_t	Turbine pressure ratio
T_i	i–th section (module) temperature ($i = 1, \cdots, 5$)
p_i	i–th section (module) pressure ($i = 1, \cdots, 5$)
m_5	5–th module mass flow rate
T_a	Ambient temperature
p_a	Ambient pressure
P_c	Compressor power
P_t	Turbine power
C_c	Compressor torque
C_t	Turbine torque
P_e	Electrical power

Simulated Application Examples (Cont'd)

Simulated Gas Turbine



$u_1(t)$, Inlet Guide Vane (IGV) angular position (α);

$u_2(t)$, fuel mass flow rate (M_f).

$y_1(t)$, pressure at the compressor inlet (p_{ic});

$y_2(t)$, pressure at the compressor outlet (p_{oc});

$y_3(t)$, pressure at the turbine outlet (p_{ot});

$y_4(t)$, temperature at the compressor outlet (T_{oc});

$y_5(t)$, temperature at the turbine outlet (T_{ot});

$y_6(t)$, electrical power at the generator terminal (P_e).

Simulated Application Examples (Cont'd)

Simulated Gas Turbine (SIMULINK®)

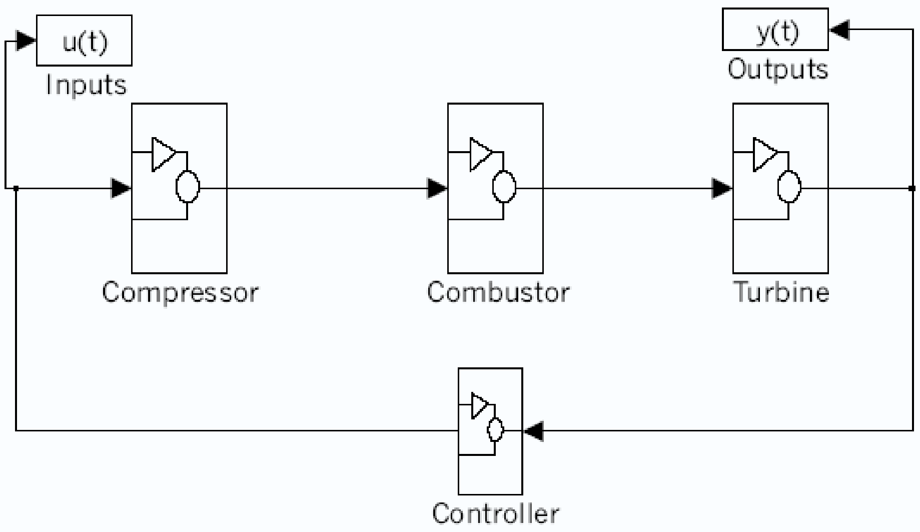


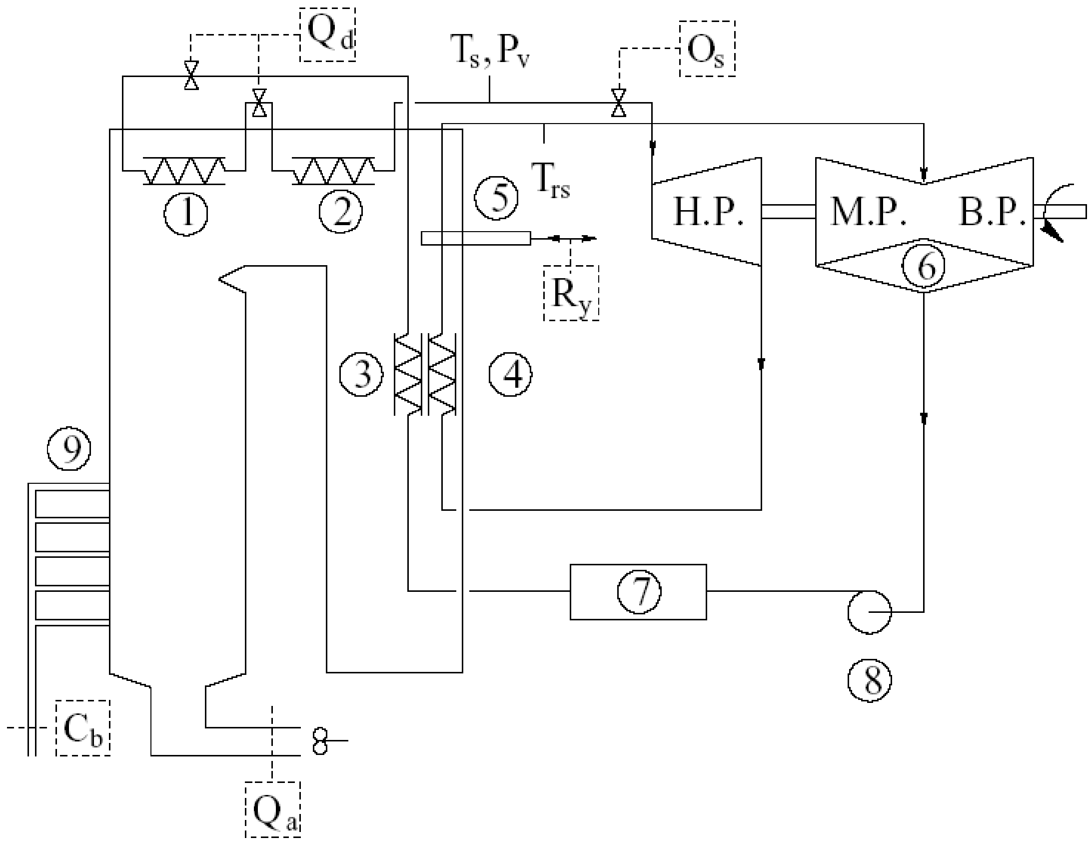
Figure 5.7: SIMULINK block diagram of the process.

Gas turbine main cycle parameters (ISO design conditions).

Air mass flow rate [kg/s]	24.4
Cycle pressure ratio (P_{oc}/P_{ic})	9.1
Electrical power (P_e) [kW]	5220
Exhaust temperature (T_{ot})[K]	796
Fuel mass flow rate (M_f) [kg/s]	0.388
IGV angle range ($\Delta\alpha$) [deg]	17

Simulated Application Examples (Cont'd)

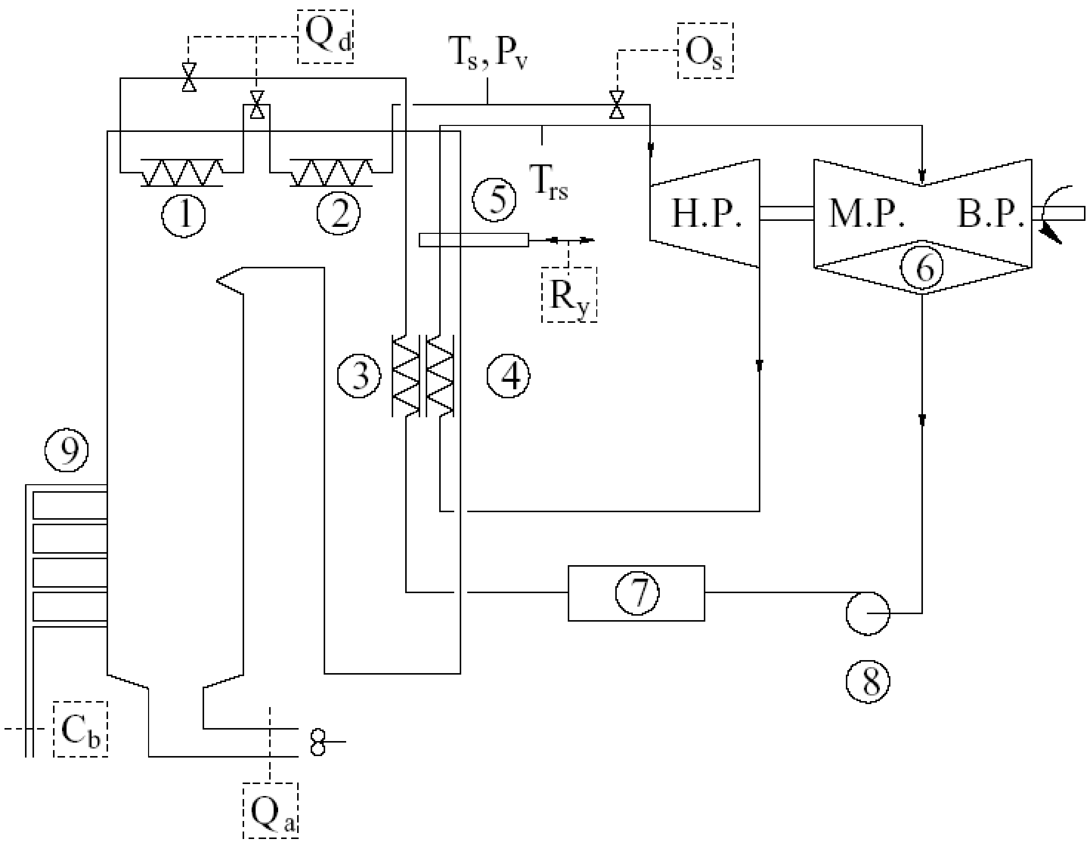
Simulated Power Plant: *Pont sur Sambre*



1. super heater (radiation);
2. super heater (convection);
3. super heater;
4. reheater;
5. dampers;
6. condenser;
7. drum;
8. water pump;
9. burner.

Simulated Application Examples (Cont'd)

Simulated Power Plant: *Pont sur Sambre*



$u_1(t)$	C_b	gas flow
$u_2(t)$	O_s	turbine valves opening
$u_3(t)$	Q_d	super heater spray flow
$u_4(t)$	R_y	gas dampers
$u_4(t)$	Q_a	air flow
$y_1(t)$	P_v	steam pressure
$y_2(t)$	T_s	main steam temperature
$y_i(t)$	T_{rs}	reheat steam temperature

Simulated Application Examples (Cont'd)

Simulated Gas Turbine

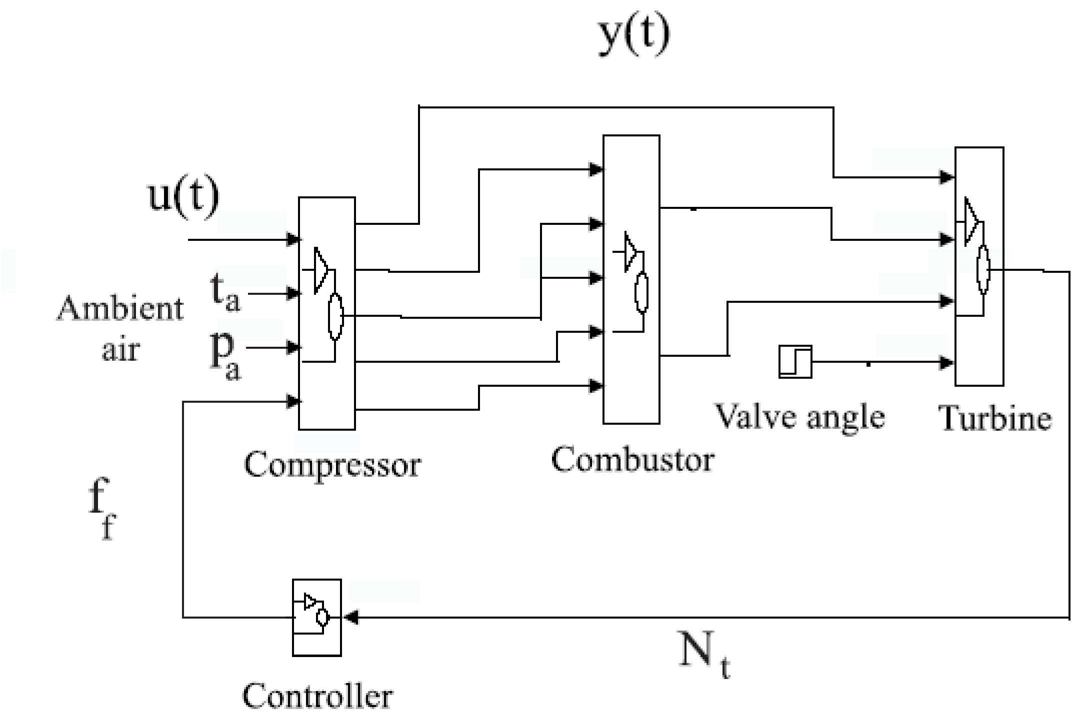


Figure 5.44: The monitored system.

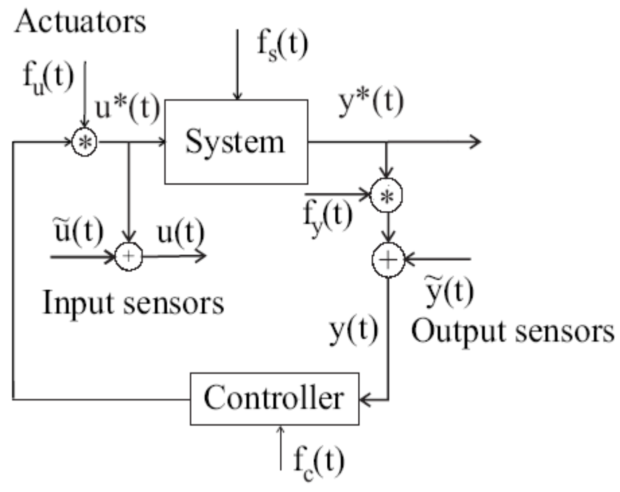


Figure 5.46: Turbine closed–loop scheme.

Simulated Application Examples (Cont'd)

Small Aircraft Model

Table 1: Nomenclature

V	True Air Speed (TAS)	H	altitude
α	angle of attack	δ_e	elevator deflection angle
β	angle of sideslip	δ_a	aileron deflection angle
P	roll rate	δ_r	rudder deflection angle
Q	pitch rate	δ_{th}	throttle aperture percentage
R	yaw rate	γ	flight path angle
ϕ	bank angle	g	acceleration of gravity
θ	elevation angle	m	airplane mass
ψ	heading angle	I_x, I_y, I_z	principal-axis inertia moments
n	engine shaft angular rate	d_t	distance of c.g. from the Thrust line

Piper Malibu

Simulated Application Examples (Cont'd)

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Simulated Application Examples (Cont'd)

6.1 SATELLITE OVERVIEW

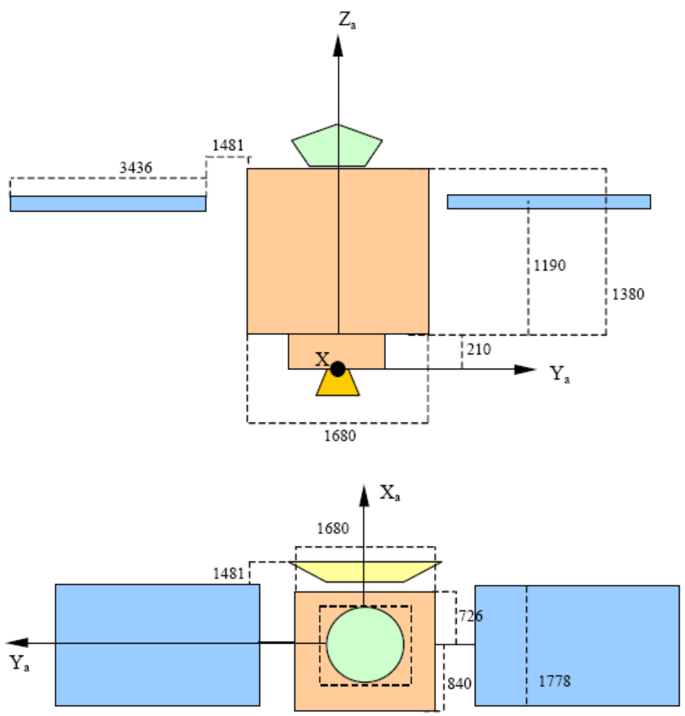


Figure 6-1: Satellite dimensions

6.4 SOLAR ARRAYS DYNAMICS

A GLOBALSTAR solar array has been selected. Its dimensions are recalled on the Figure 6-2.

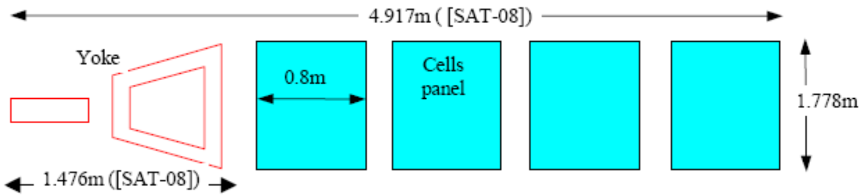


Figure 6-2 : MARS-EXPRESS solar array

Simple SA model

Aerospace Satellite

Simulated Application Examples (Cont'd)

Aerospace Satellite

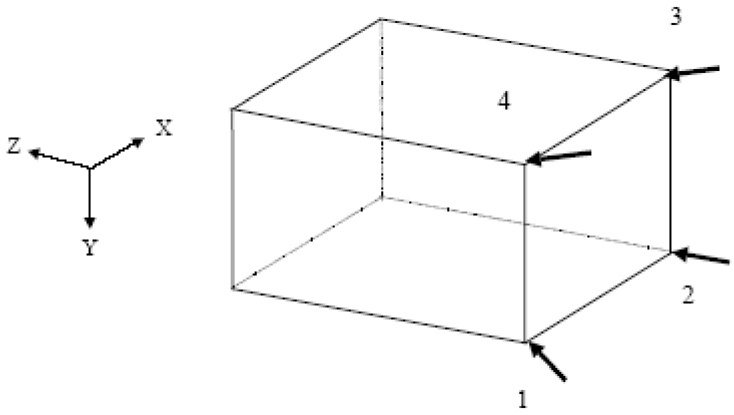


Figure 6-3 : Thrusters implementation

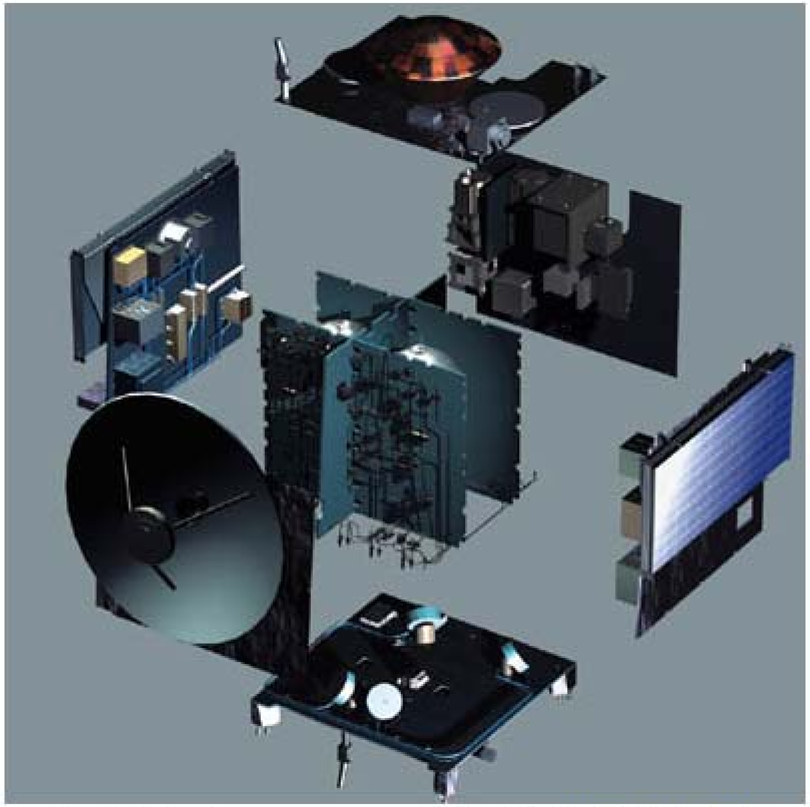
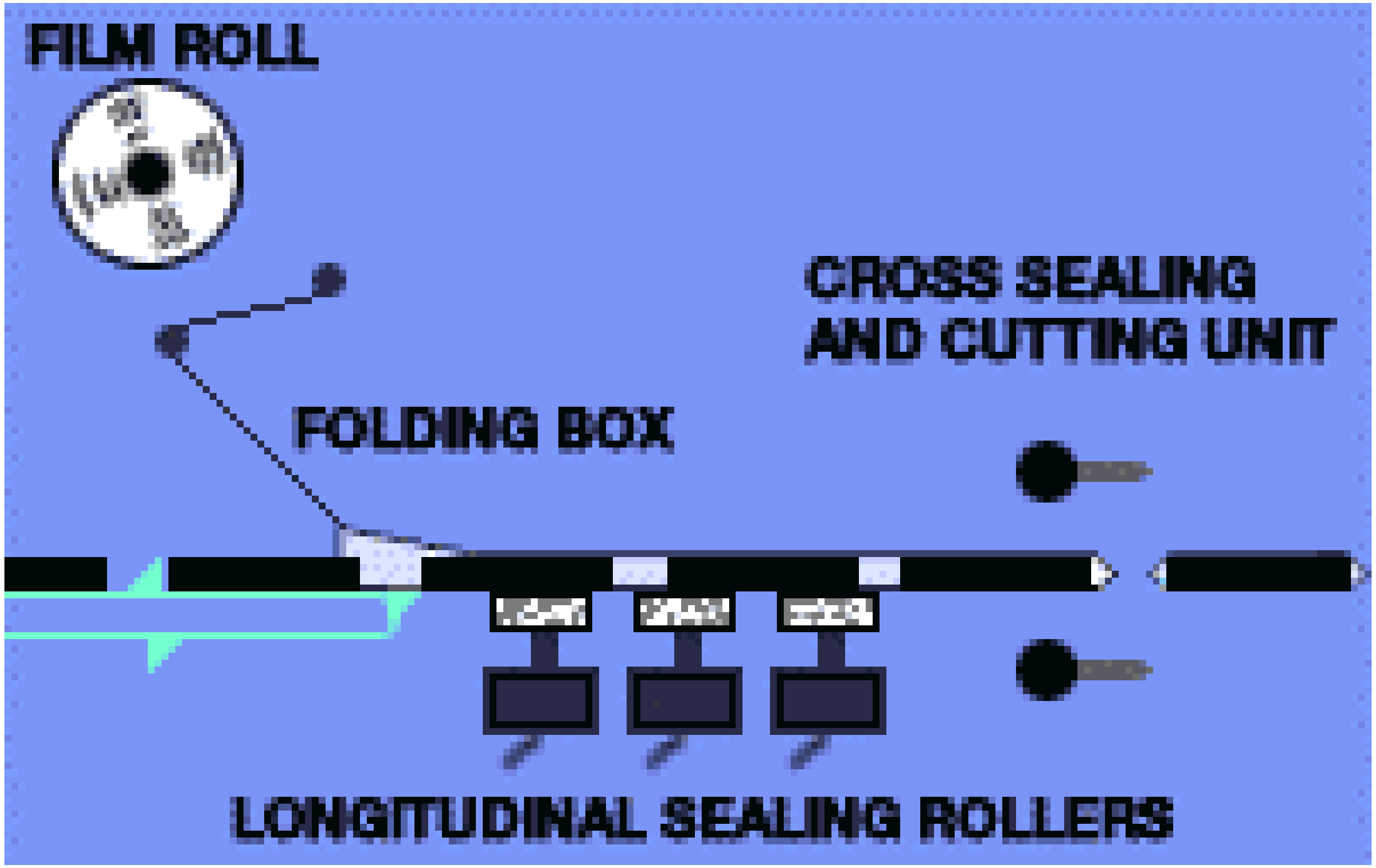


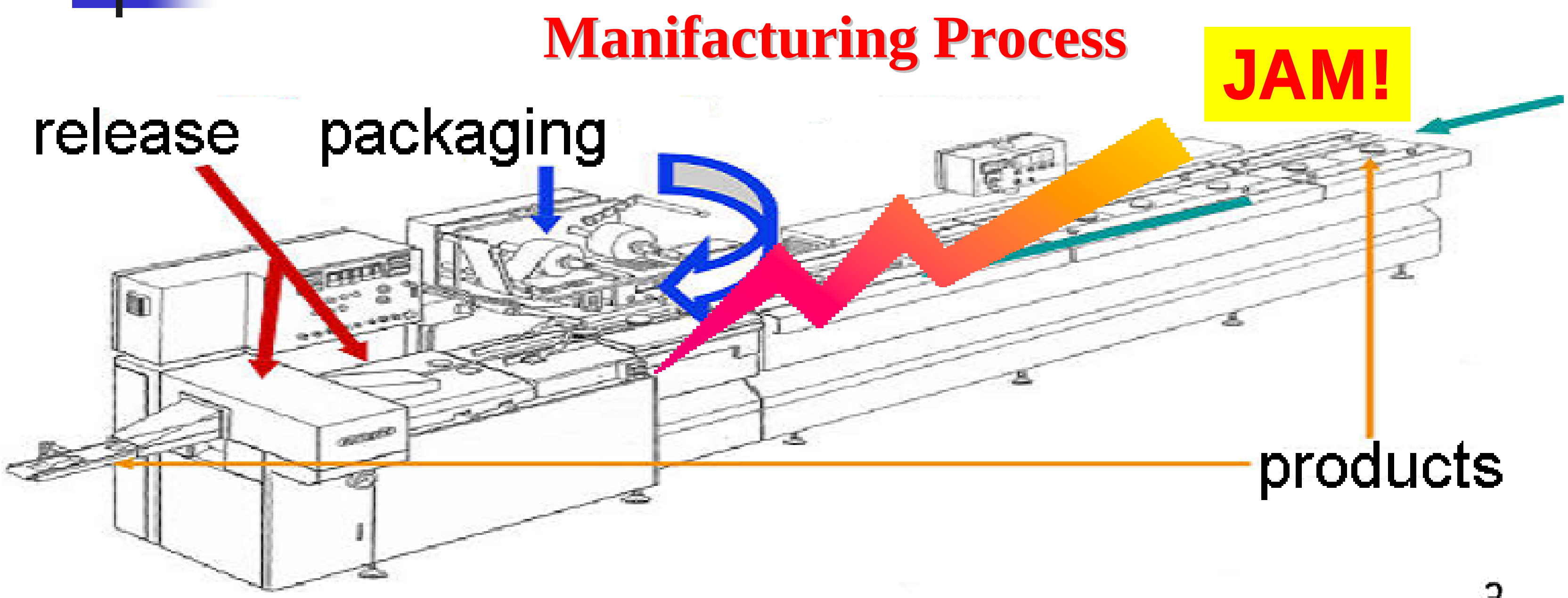
Fig. 1: Mars Express spacecraft (MEX) (www.esa.int)

Simulated Application Examples (Cont'd)

Manifacturing Process



Simulated Application Examples (Cont'd)



(possibly unskilled) human operator

Introduction

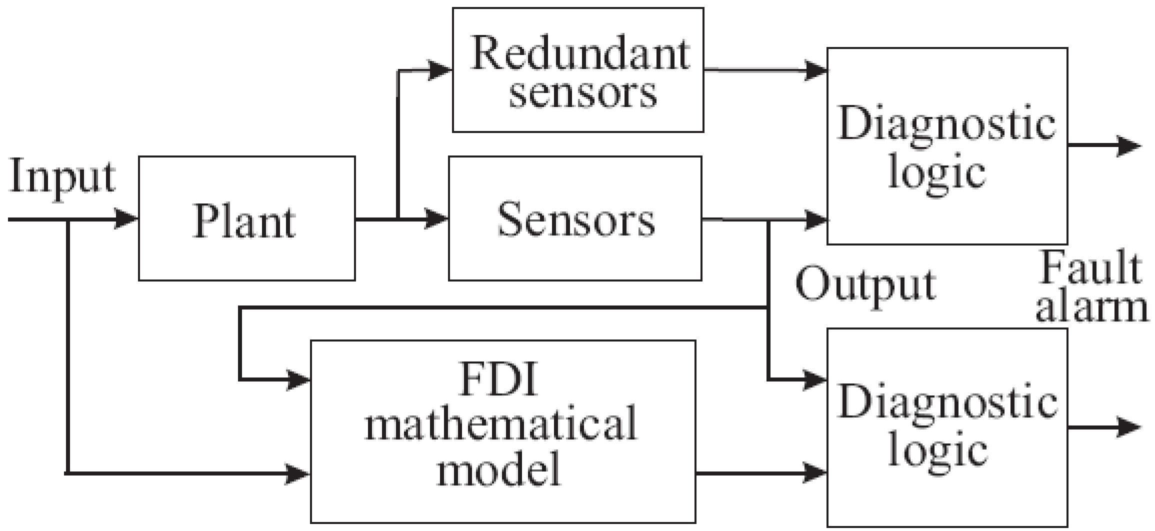


Figure 1.1: Comparison between hardware and analytical redundancy schemes.

Introduction (Cont'd)

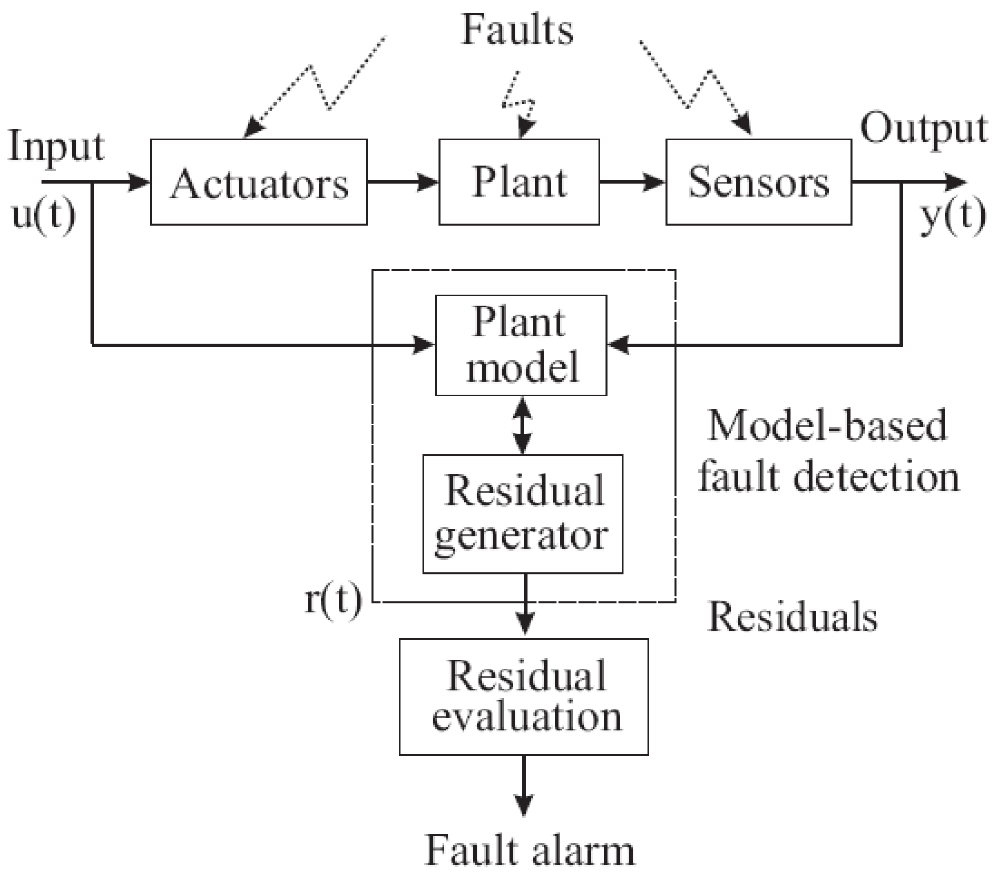


Figure 1.2: Scheme for the model–based fault detection.

Introduction (Cont'd)

- Model-Based FDI Methods:

1. Output observers (OO, estimators, filters);
2. Parity equations;
3. Identification and parameter estimation.

Introduction (Cont'd)

- Signal Model-Based Methods:

  1. Bandpass filters;
  2. Spectral analysis (FFT);
  3. Maximum-entropy estimation.

- Change Detection: Residual Analysis

  1. Mean and variance estimation;
  2. Likelihood-ratio test, Bayes decision;
  3. Run-sum test.

Introduction (Cont'd)

- Model Uncertainty and FDI
  - Model-reality mismatch
  - Sensitivity problem: incipient faults!
- Robustness in FDI
  - Disturbance, modelling errors, uncertainty
  - UIO and Kalman filter: robust residual generation
- System Identification for FDI
  - Estimation of a reliable model
  - Modelling accuracy
  - Disturbance estimation (recall: ARX, ARMAX, BJ)

Introduction (Cont'd)

- Fault Identification Methods
  - Fault nature (type, shape) & size (amplitude)

    1. Geometrical distance and probabilistic methods;
    2. Artificial neural networks;
    3. Fuzzy clustering.

- Approximate Reasoning Methods:

    1. Probabilistic reasoning;
    2. Possibilistic reasoning with fuzzy logic;
    3. Reasoning with artificial neural networks.

Introduction (Cont'd)

- FDI applications status & review

Table 1.1: FDI applications and number of contributions.

Application	Number of contributions
Simulation of real processes	55
Large-scale pilot processes	44
Small-scale laboratory processes	18
Full-scale industrial processes	48

Table 1.2: Fault type and number of contributions.

Fault type	Number of contributions
Sensor faults	69
Actuator faults	51
Process faults	83
Control loop or controller faults	8

Introduction (Cont'd)

- FDI applications status & review

Table 1.3: FDI methods and number of contributions.

Method type	Number of contributions
Observer	53
Parity space	14
Parameter estimation	51
Frequency spectral analysis	7
Neural networks	9

Table 1.4: Residual evaluation methods and number of contributions.

Evaluation method	Number of contributions
Neural networks	19
Fuzzy logic	5
Bayes classification	4
Hypothesis testing	8

Introduction (Cont'd)

- FDI applications status & review

Table 1.5: Reasoning strategies and number of contributions.

Reasoning strategy	Number of contributions
Rule based	10
Sign directed graph	3
Fault symptom tree	2
Fuzzy logic	6

Table 1.6: Applications of model–based fault detection.

FDD	Number of contributions
Milling and grinding processes	41
Power plants and thermal processes	46
Fluid dynamic processes	17
Combustion engine and turbines	36
Automotive	8
Inverted pendulum	33
Miscellaneous	42
DC motors	61
Stirred tank reactor	27
Navigation system	25
Nuclear process	10

Model-based FDI Techniques

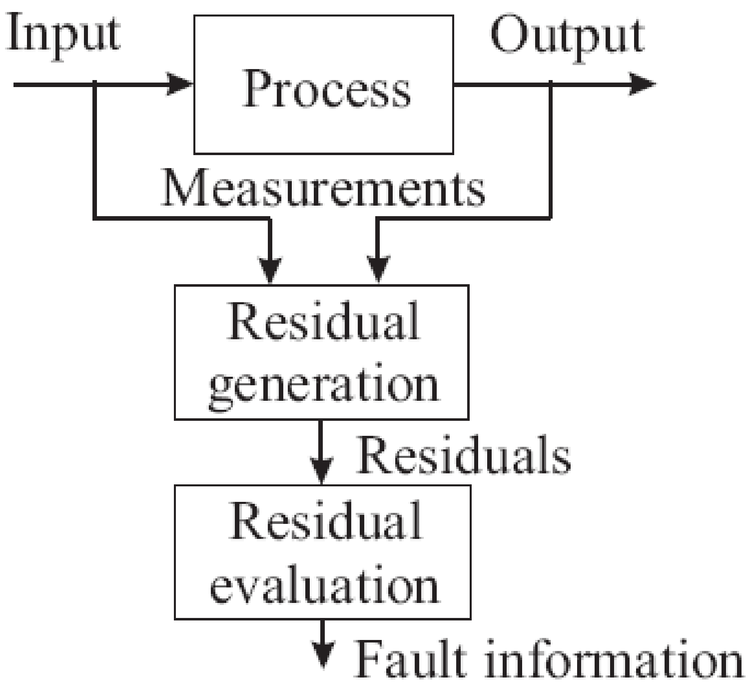


Figure 2.1: Structure of model-based FDI system.

Model-based FDI Techniques (Cont'd)

1. **Residual generation:** this block generates residual signals using available inputs and outputs from the monitored system. This residual (or fault symptom) should indicate that a fault has occurred. It should normally be zero or close to zero under no fault condition, whilst distinguishably different from zero when a fault occurs. This means that the residual is characteristically independent of process inputs and outputs, in ideal conditions. Referring to Figure 2.1, this block is called *residual generation*.

2. **Residual evaluation:** This block examines residuals for the likelihood of faults and a decision rule is then applied to determine if any faults have occurred. The *residual evaluation* block, shown in Figure 2.1, may perform a simple threshold test (geometrical methods) on the instantaneous values or moving averages of the residuals. On the other hand, it may consist of statistical methods, *e.g.*, generalised likelihood ratio testing or sequential probability ratio testing [Isermann, 1997, Willsky, 1976, Basseville, 1988, Patton et al., 2000].

Model-based FDI Techniques (Cont'd)

➢ Modelling of Faulty Systems

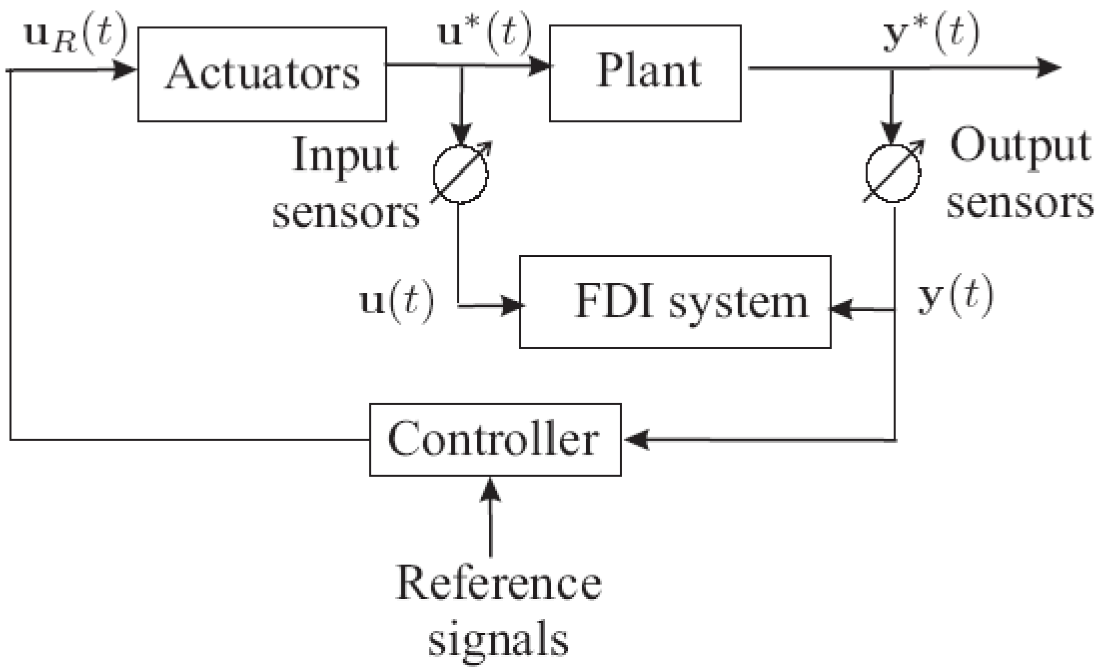


Figure 2.2: Fault diagnosis in a closed-loop system.

Model-based FDI Techniques (Cont'd)

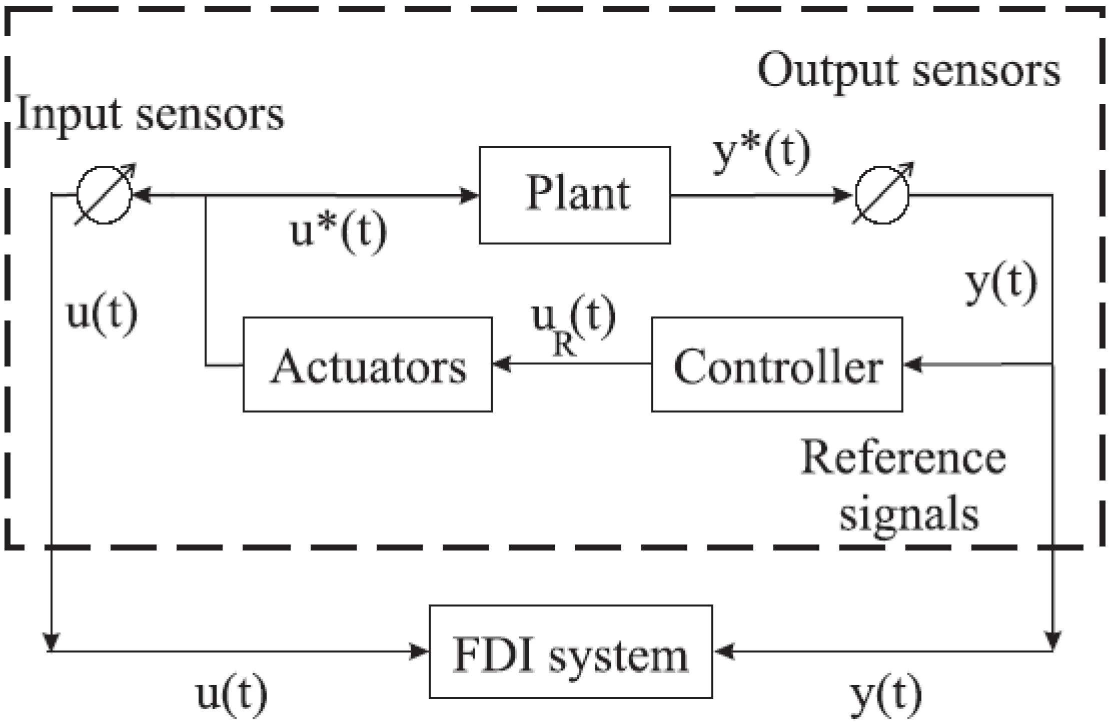


Figure 2.3: The rearranged fault diagnosis scheme.

Model-based FDI Techniques (Cont'd)

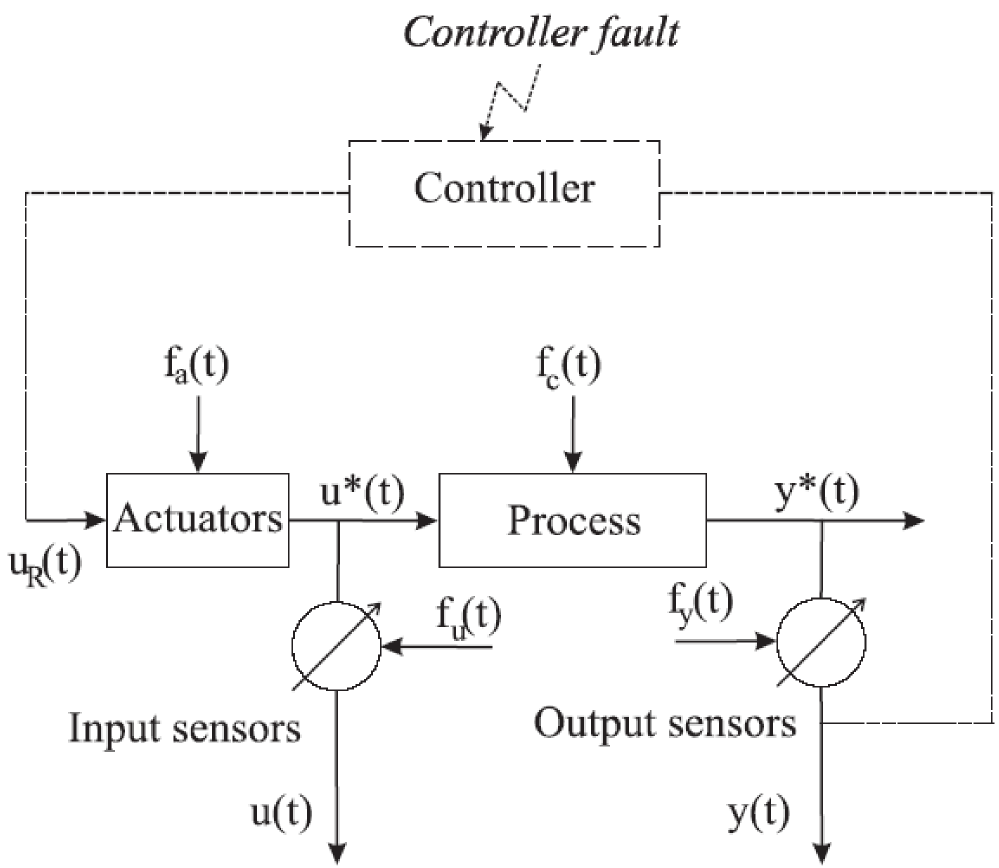


Figure 2.4: The controlled system and fault topology.

Model-based FDI Techniques (Cont'd)

Fault Location:

- Actuators
- Process or system components
- Input sensors
- Output sensors
- Controllers

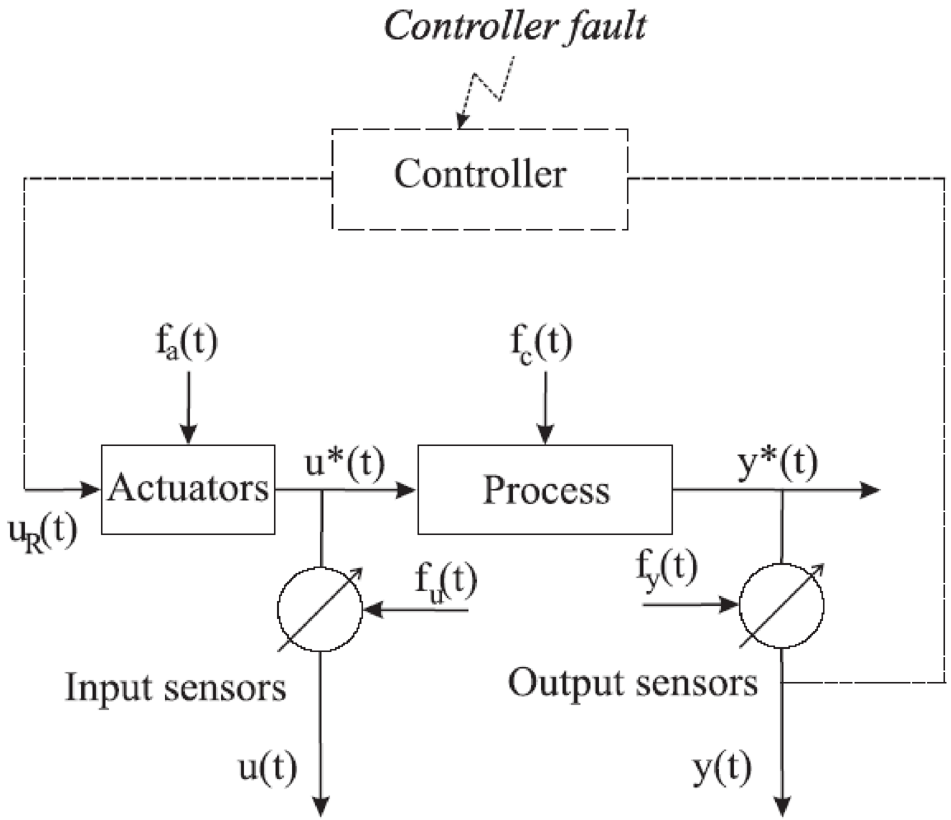


Figure 2.4: The controlled system and fault topology.

Model-based FDI Techniques (Cont'd)

Fault and System Modelling

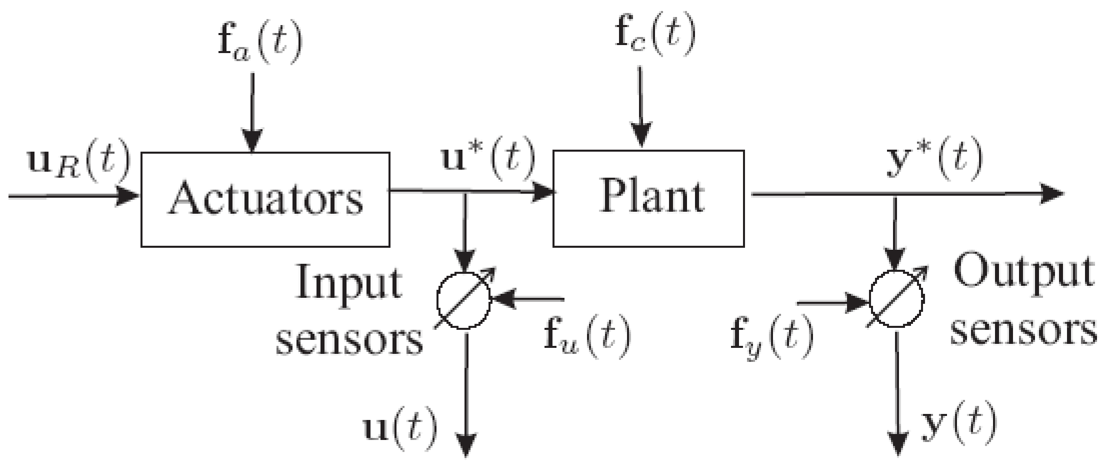


Figure 2.5: The monitored system and fault topology.

$$\begin{cases} \mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}^*(t) \\ \mathbf{y}^*(t) = \mathbf{C}\mathbf{x}(t) \end{cases}$$

Model-based FDI Techniques (Cont'd)

Fault and System Modelling

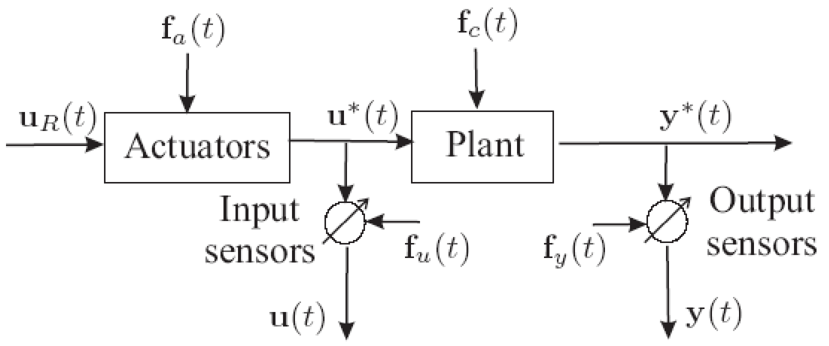


Figure 2.5: The monitored system and fault topology.

$$\begin{cases} \mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}^*(t) \\ \mathbf{y}^*(t) & = & \mathbf{C}\mathbf{x}(t) \end{cases}$$

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}^*(t) + \mathbf{f}_c(t)$$

$$\mathbf{f}_c(t) = I_i \Delta a_{ij} x_j(t)$$

$$\begin{cases} \mathbf{u}(t) & = & \mathbf{u}^*(t) + \mathbf{f}_u(t) \\ \mathbf{y}(t) & = & \mathbf{y}^*(t) + \mathbf{f}_y(t) \end{cases}$$

Model-based FDI Techniques (Cont'd)

Fault and System Modelling

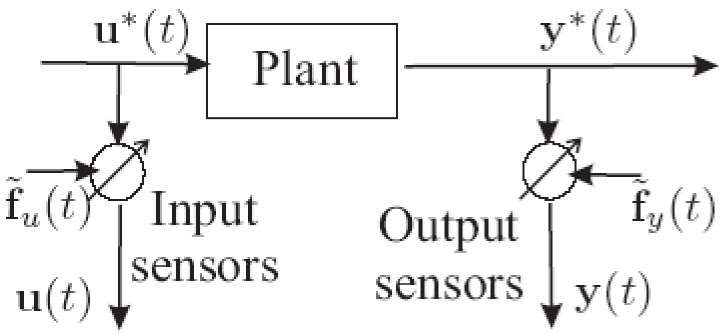


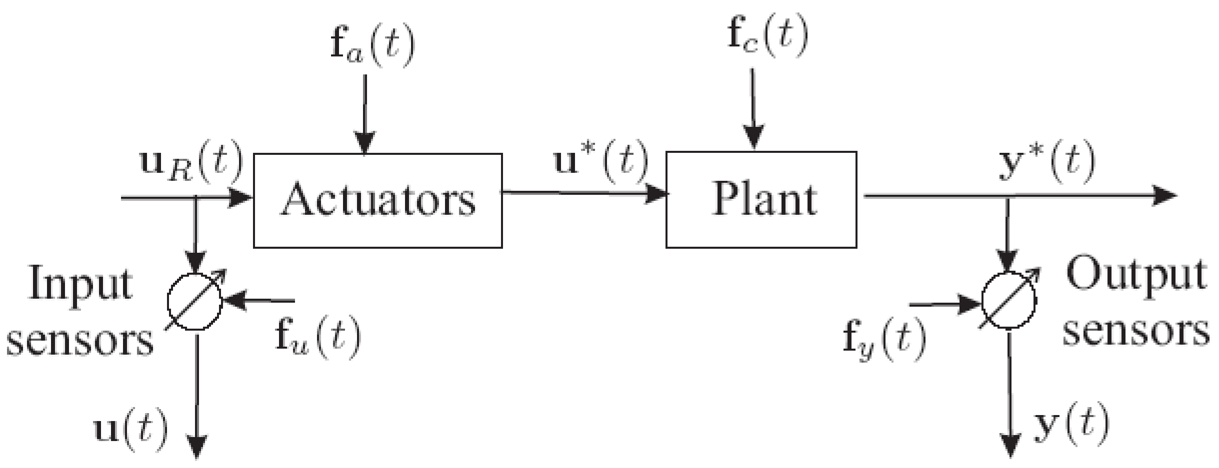
Figure 2.6: The structure of the plant sensors.

$$\begin{cases} \mathbf{u}(t) = \mathbf{u}^*(t) + \tilde{\mathbf{u}}(t) \\ \mathbf{y}(t) = \mathbf{y}^*(t) + \tilde{\mathbf{y}}(t) \end{cases}$$

$$\begin{cases} \mathbf{u}(t) = \mathbf{u}^*(t) + \tilde{\mathbf{u}}(t) + \mathbf{f}_u(t) \\ \mathbf{y}(t) = \mathbf{y}^*(t) + \tilde{\mathbf{y}}(t) + \mathbf{f}_y(t) \end{cases}$$

Model-based FDI Techniques (Cont'd)

Fault and System Modelling



$$\mathbf{u}^*(t) = \mathbf{u}_R(t) + \mathbf{f}_a(t)$$

$$\begin{cases} \mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t) + \mathbf{f}_c(t) + B\mathbf{u}^*(t) \\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t) + \mathbf{f}_y(t) \\ \mathbf{u}(t) & = & \mathbf{u}^*(t) + \mathbf{f}_u(t) \end{cases}$$

Model-based FDI Techniques (Cont'd)

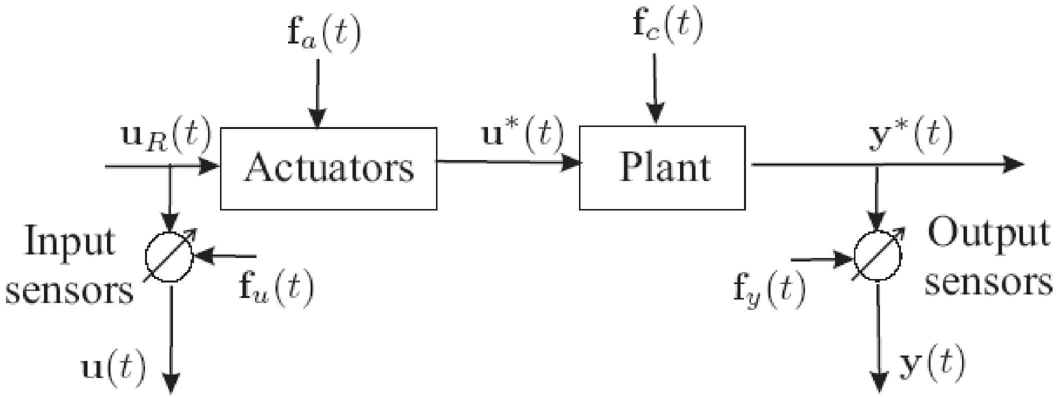


Figure 2.7: Fault topology with actuator input signal measurement.

- Modelling of Faulty Systems

$$\mathbf{f}(t) = [\mathbf{f}_a^T,\ \mathbf{f}_u^T,\ \mathbf{f}_c^T,\ \mathbf{f}_y^T]^T \in \Re^k$$

$$\begin{cases} \mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t) + \mathbf{f}_c(t) + \mathbf{B}\mathbf{f}_a(t) + \mathbf{B}\mathbf{u}^*(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{f}_y(t) \\ \mathbf{u}(t) &= \mathbf{u}^*(t) + \mathbf{f}_u(t) \end{cases}$$

$$\begin{cases} \mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}^*(t) + \mathbf{L}_1\mathbf{f}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{L}_2\mathbf{f}(t) \\ \mathbf{u}(t) &= \mathbf{u}^*(t) + \mathbf{L}_3\mathbf{f}(t) \end{cases}$$

Model-based FDI Techniques (Cont'd)

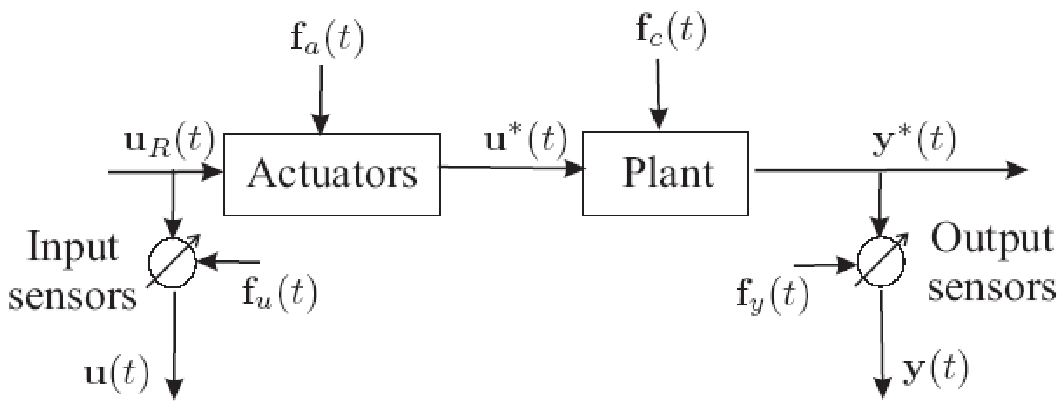


Figure 2.7: Fault topology with actuator input signal measurement.

- Modelling of Faulty Systems

$$\mathbf{y}(z) = \mathbf{G}_{yu^*}(z)\mathbf{u}^*(z) + \mathbf{G}_{yf}(z)\mathbf{f}(z)$$

- Transfer function description:

$$\begin{cases} \mathbf{G}_{yu^*}(z) & = & \mathbf{C}(zI - \mathbf{A})^{-1}\mathbf{B} \\ \mathbf{G}_{yf}(z) & = & \mathbf{C}(zI - \mathbf{A})^{-1}\mathbf{L}_1 + \mathbf{L}_2 \end{cases}$$

Residual Generator Structure

$$\left\{ \begin{array}{lcll} \mathbf{z}(t) & = & W_1\big(\mathbf{u}(\cdot), \mathbf{y}(\cdot)\big) & \\ \mathbf{r}(t) & = & W_2\big(\mathbf{z}(\cdot), \mathbf{y}(\cdot)\big) & = \ 0. \end{array} \right.$$

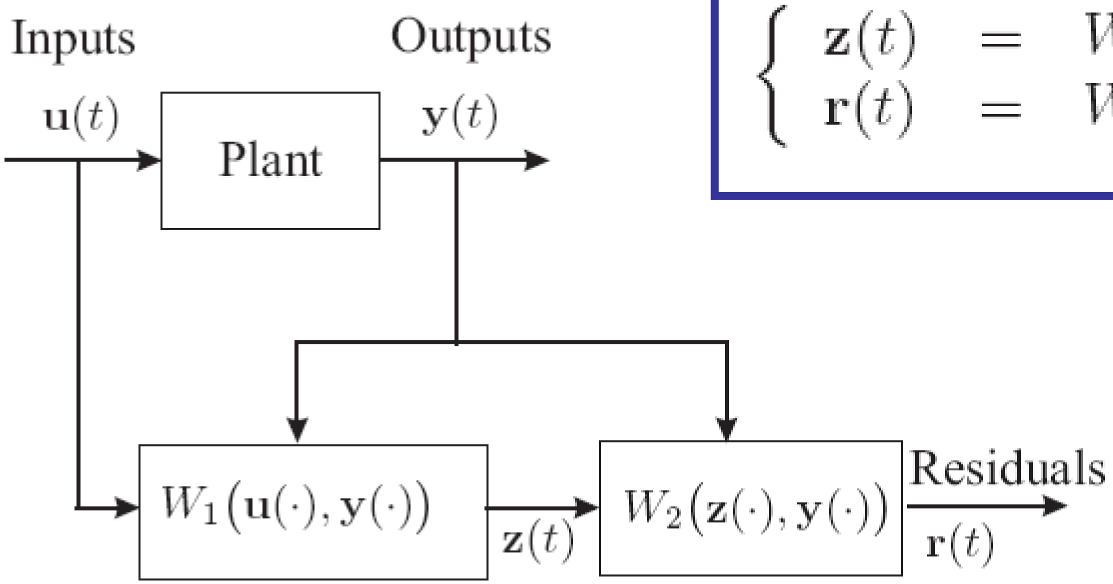


Figure 2.8: Residual generator general structure.

Under fault-free assumptions, the residual signal *r(t)* is “almost zero”

Residual General Structure (Cont’d)

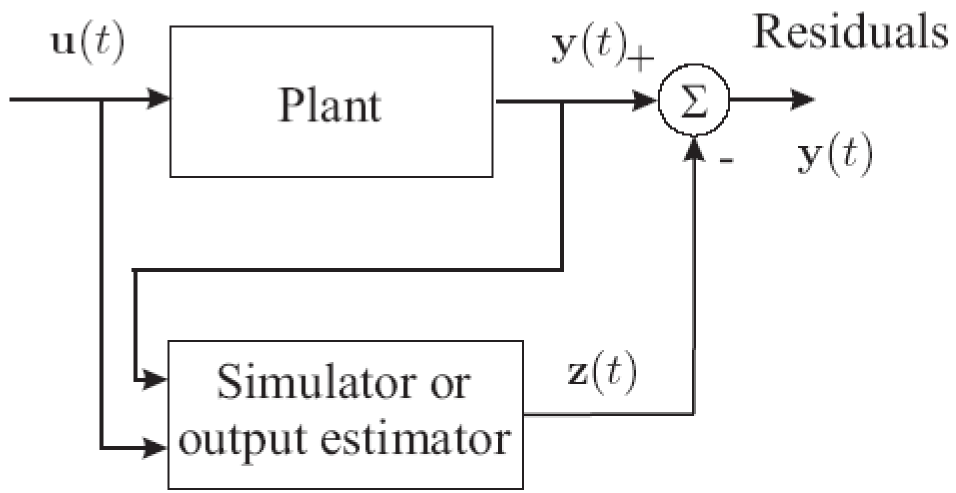


Figure 2.9: Residual generation via system simulator.

Residual generation via system simulator

$$\boxed{\mathbf{r}(t) = \mathbf{z}(t) - \mathbf{y}(t)}$$

z(t) is the simulated plant output

Residual General Structure (Cont'd)

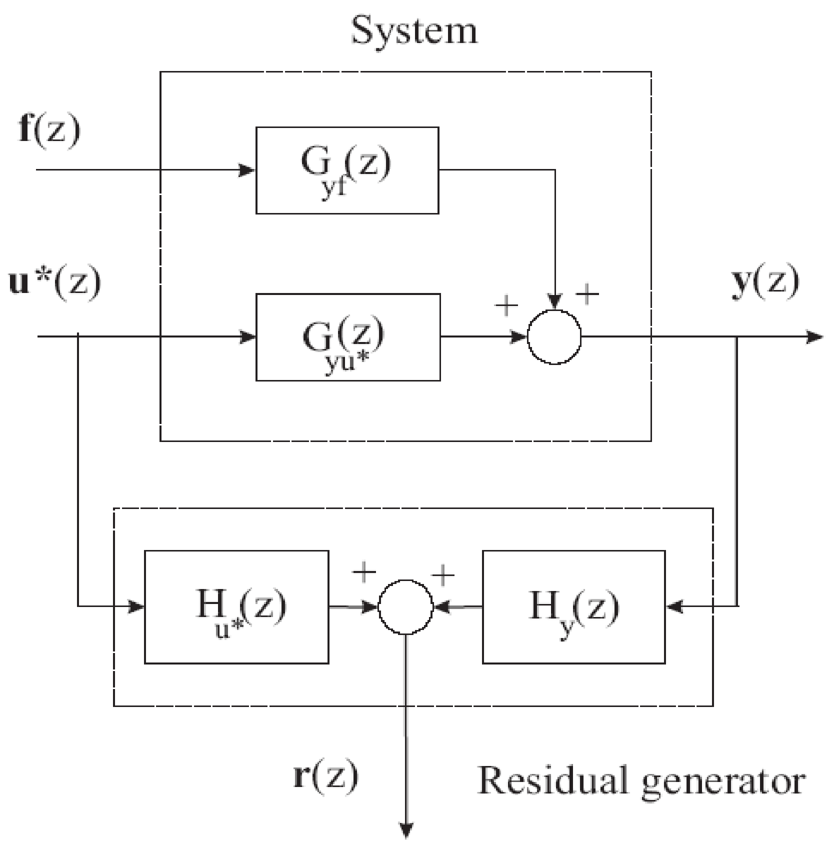


Figure 2.10: Residual generator general structure.

$$\mathbf{y}(z) = \mathbf{G}_{yu^*}(z)\mathbf{u}^*(z) + \mathbf{G}_{yf}(z)\mathbf{f}(z)$$

Residual generator:

$$\mathbf{r}(z) = \begin{bmatrix} \mathbf{H}_{u^*}(z) & \mathbf{H}_y(z) \end{bmatrix} \begin{bmatrix} \mathbf{u}^*(z) \\ \mathbf{y}(z) \end{bmatrix} =$$

$$= H_{u^*}(z)\mathbf{u}^*(z) + \mathbf{H}_y(z)\mathbf{y}(z)$$

$$\mathbf{r}(t) = \mathbf{0} \text{ if and only if } \mathbf{f}(t) = \mathbf{0}$$

Constraint conditions: ***design***

$$\boxed{\mathbf{H}_{u^*}(z) + \mathbf{H}_y(z)\mathbf{G}_{yu^*} = \mathbf{0}}$$

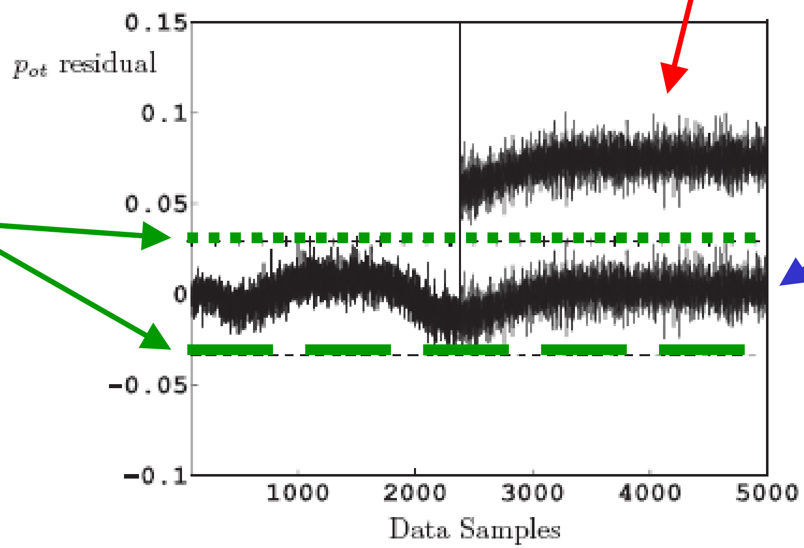
General Residual Evaluation

$$\begin{cases} J\left(\mathbf{r}(t)\right) \le \varepsilon(t) & \text{for} \quad \mathbf{f}(t) = \mathbf{0} \\ J\left(\mathbf{r}(t)\right) > \varepsilon(t) & \text{for} \quad \mathbf{f}(t) \neq \mathbf{0} \end{cases}$$

Faulty residual

Detection thresholds

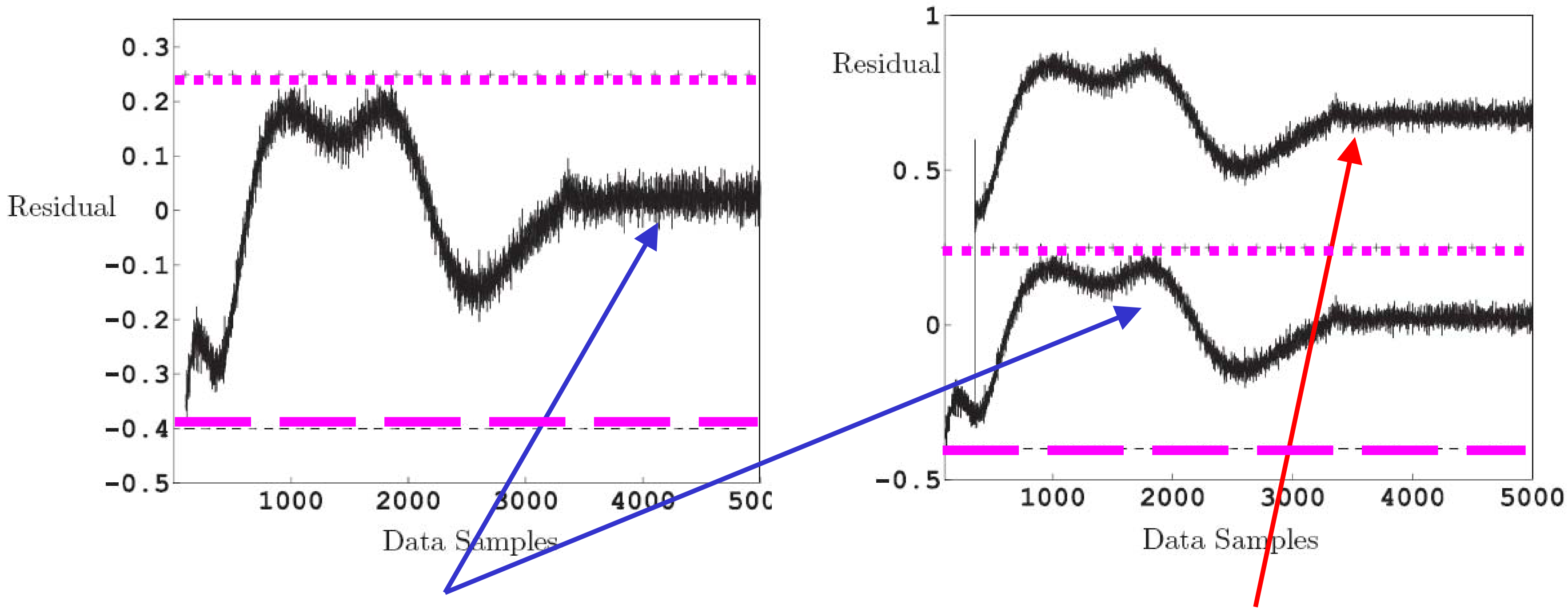
ε(t)



Fault free residual

General Residual Evaluation *(example)*

Detection thresholds



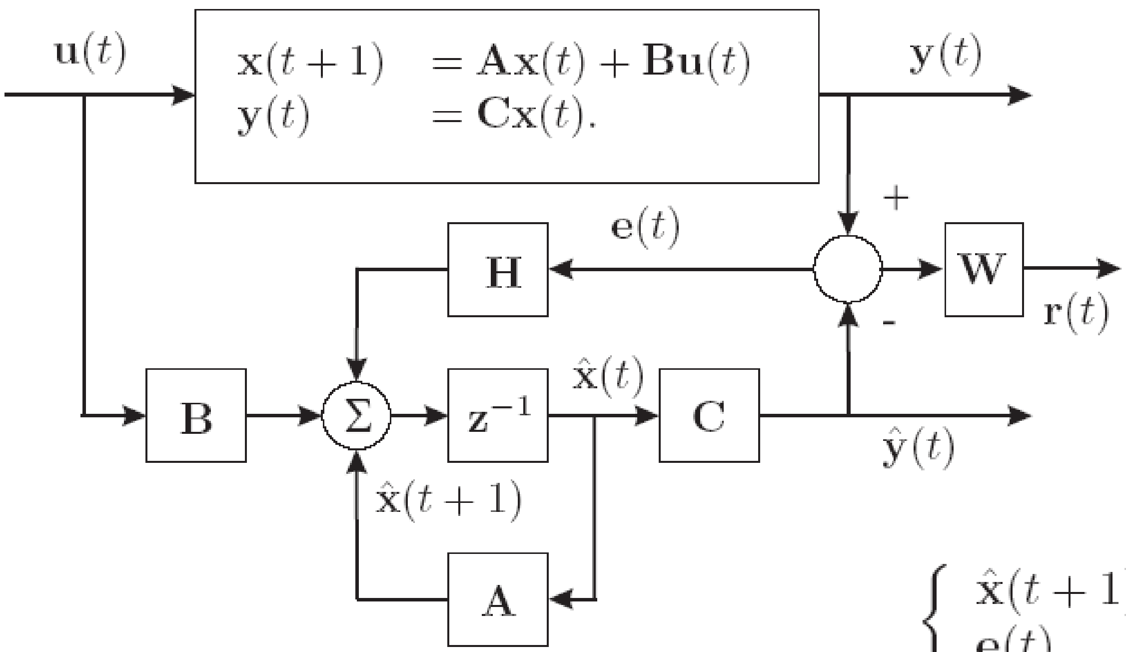
Fault free residual **Fault-free & faulty residuals**

Residual Generation Techniques

- Fault detection via parameter estimation
  - Observer-based appoaches
  - Parity (vector) relations

Residual General Structure

- Observer-based approach



Plant model

$$\begin{cases} \mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t). \end{cases}$$

Observer model

$$\begin{cases} \hat{\mathbf{x}}(t+1) & = & \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{H}\mathbf{e}(t) \\ \mathbf{e}(t) & = & \mathbf{y}(t) - \mathbf{C}\hat{\mathbf{x}}(t). \end{cases}$$

Output estimation approach!

Residual Generator Structure

Plant model

$$\begin{cases} \mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t). \end{cases}$$

Observer model

$$\begin{cases} \hat{\mathbf{x}}(t+1) & = & \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{H}\mathbf{e}(t) \\ \mathbf{e}(t) & = & \mathbf{y}(t) - \mathbf{C}\hat{\mathbf{x}}(t). \end{cases}$$

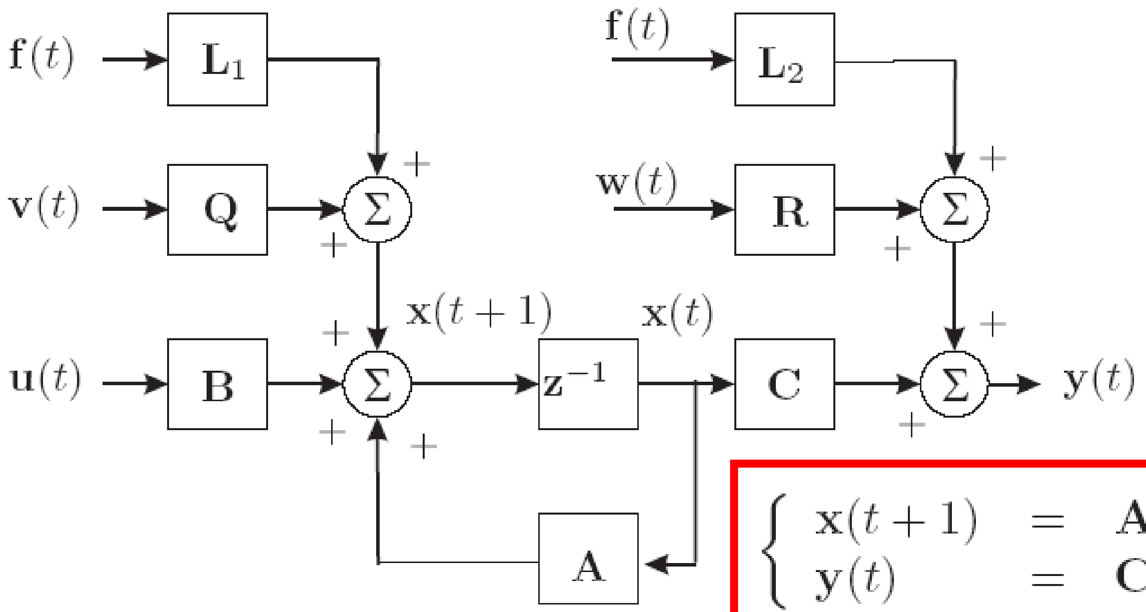
State estimation model

$$\begin{cases} \mathbf{e}_x(t) & = & \mathbf{x}(t) - \hat{\mathbf{x}}(t) \\ \mathbf{e}_x(t+1) & = & (\mathbf{A} - \mathbf{H}\mathbf{C})\mathbf{e}_x(t). \end{cases}$$

State estimation property $\lim_{t \to \infty} \mathbf{e}_x(t) = \mathbf{0}$ *(fault-free case!!!)*

Residual Generator Property

+ disturbance signals and fault



$$\begin{cases} \mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{Q}\mathbf{v}(t) + \mathbf{L}_1\mathbf{f}(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{R}\mathbf{w}(t) + \mathbf{L}_2\mathbf{f}(t) \end{cases}$$

Figure 2.14: MIMO process with faults and noises.

Residual Generator Property (Cont'd)

+ fault signals

System model

$$\begin{cases} \mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{Q}\mathbf{v}(t) + \mathbf{L}_1\mathbf{f}(t) \\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t) + \mathbf{R}\mathbf{w}(t) + \mathbf{L}_2\mathbf{f}(t) \end{cases}$$

Observer model

$$\mathbf{e}_x(t+1) = (\mathbf{A} - \mathbf{H}\mathbf{C})\mathbf{e}_x(t) + \mathbf{L}_1\mathbf{f}(t) - \mathbf{H}\mathbf{L}_2\mathbf{f}(t)$$

Output estimation error with faults *but noise-free*

$$\boxed{\mathbf{e}(t) = \mathbf{C}\mathbf{e}_x(t) + \mathbf{L}_2\mathbf{f}(t).}$$

Both *e(t)* and e_x(t) are suitable residuals!

Output Observer & UIO: *intro*

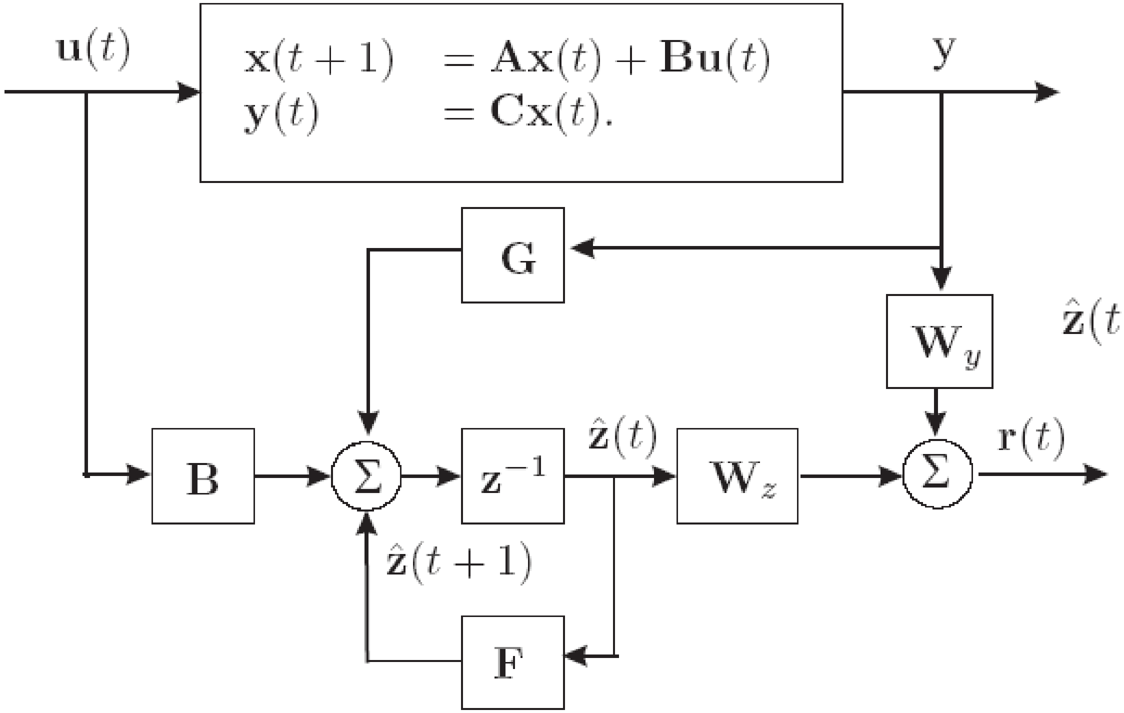


Figure 2.15: Process and output observer.

State transformation

$$\mathbf{z}(t) = \mathbf{T}\mathbf{x}(t)$$

Observer

$$\hat{\mathbf{z}}(t+1) = \mathbf{F}\hat{\mathbf{z}}(t) + \mathbf{J}\mathbf{u}(t) + \mathbf{G}\mathbf{y}(t)$$

Residual

$$\mathbf{r}(t) = \mathbf{W}_z\hat{\mathbf{z}}(t) + \mathbf{W}_y\mathbf{y}(t)$$

State estimation error

$$\mathbf{e}_x(t) = \hat{\mathbf{z}}(t) - \mathbf{z}(t) = \hat{\mathbf{z}}(t) - \mathbf{T}\mathbf{x}(t)$$

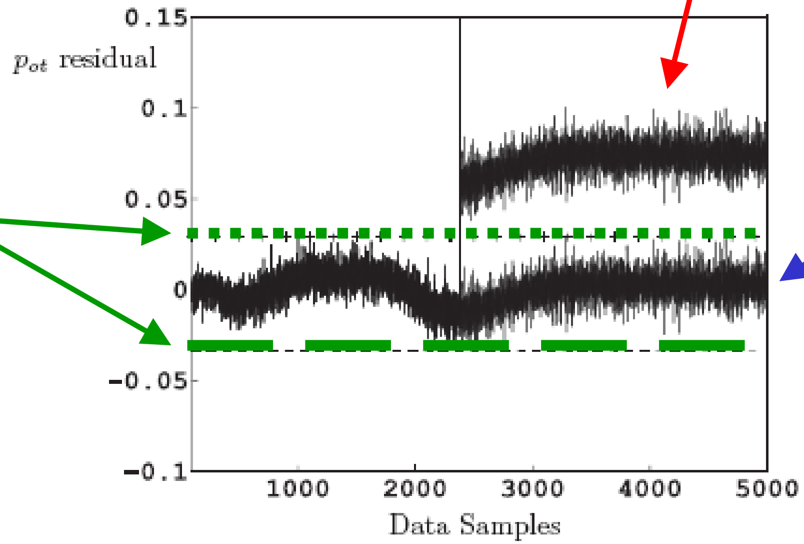
General Residual Evaluation

$$\begin{cases} J\left(\mathbf{r}(t)\right) \le \varepsilon(t) & \text{for} \quad \mathbf{f}(t) = \mathbf{0} \\ J\left(\mathbf{r}(t)\right) > \varepsilon(t) & \text{for} \quad \mathbf{f}(t) \neq \mathbf{0} \end{cases}$$

Faulty residual

Fault free residual

Detection thresholds
ε(t)



Change Detection & Residual Evaluation

$$\begin{cases} J(\mathbf{r}(t)) \leq \varepsilon(t) & \text{for} \quad \mathbf{f}(t) = \mathbf{0} \\ J(\mathbf{r}(t)) > \varepsilon(t) & \text{for} \quad \mathbf{f}(t) \neq \mathbf{0} \end{cases}$$

$$\boxed{J\big(r(t)\big) \equiv |r(t)|}$$

Detection thresholds

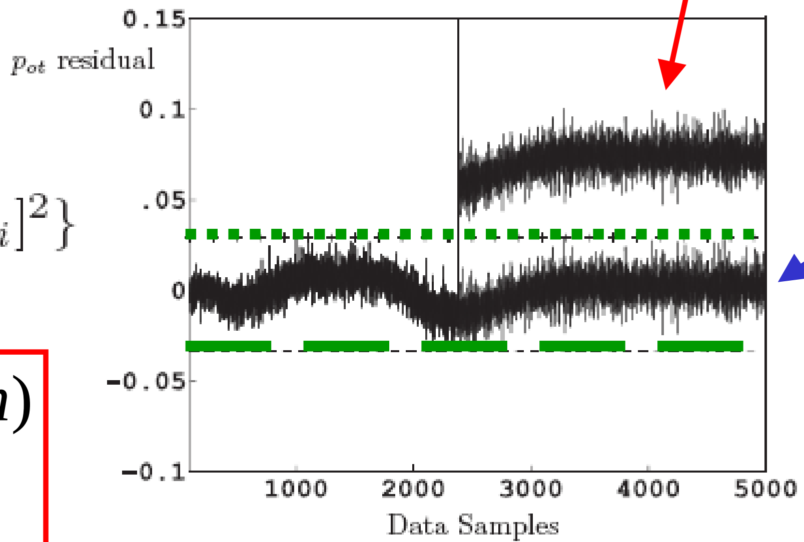
$\varepsilon(t)$

$$\bar{r}_i = E\{r_i(t)\}; \qquad \bar{\sigma}_i^2 = E\{[r_i(t) - \bar{r}_i]^2\}$$

$$\boxed{\begin{array}{l} \varepsilon(t) = \bar{r}_i \pm \delta \times \bar{\sigma}_i \qquad (i = 1, \cdots, m) \\ \text{with} \quad \delta \geq 3 \end{array}}$$

Faulty residual

Fault free residual



Residual Generation

- ✓ Output Observers
- ➢ Unknown Input Observers
- ✓ Fault Detection
  - ➢ Fault Isolation, *i.e. where is the fault*?

Output Observer

Process model with faults

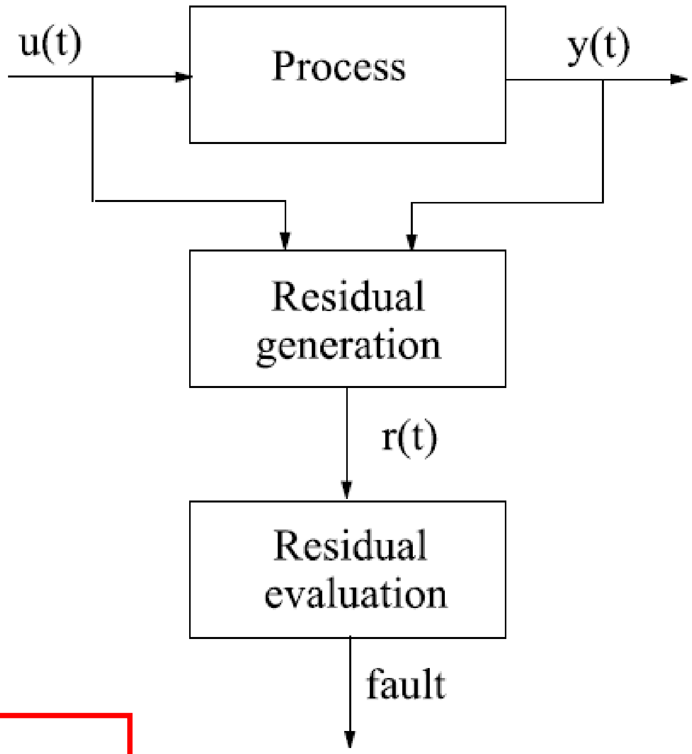
$$\begin{aligned} \mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}(\mathbf{u}(t) + \mathbf{f}_c(t)) + \mathbf{f}_s(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t), \end{aligned}$$

Input-output sensor faults

$$\left.\begin{aligned} \mathbf{u}(t) &= \mathbf{f}_u(t) + \mathbf{u}^*(t) \\ \mathbf{y}(t) &= \mathbf{f}_y(t) + \mathbf{y}^*(t) \end{aligned}\right\}$$

Observer for the i-th output $y_i(t)$

$$\boxed{\mathbf{x}^i(t+1) = \mathbf{A}_i \mathbf{x}^i(t) + \mathbf{B}_i \mathbf{u}(t) + \mathbf{K}_i \left(y_i(t) - \mathbf{C}_i \mathbf{x}^i(t)\right)}$$



(A_i, B_i, C_i) is the state – space process model

Output Observer for *Fault Detection*

Given the observer model

$$\mathbf{x}^i(t+1) = \mathbf{A}_i \mathbf{x}^i(t) + \mathbf{B}_i \mathbf{u}(t) + \mathbf{K}_i \left(y_i(t) - \mathbf{C}_i \mathbf{x}^i(t) \right)$$

Under fault-free assumptions

$r_i(t) = y_i^*(t) - \hat{y}_i(t) = \mathbf{C}_i \left(\mathbf{x}_i(t) - \mathbf{x}^i(t) \right)$ is equal to zero.

Fault detection logic

fixed threshold ϵ,

$$\left. \begin{array}{lll} \mathbf{r}(t) \leq \epsilon & \text{for} & \mathbf{f}(t) = \mathbf{0} \\ \mathbf{r}(t) > \epsilon & \text{for} & \mathbf{f}(t) \neq \mathbf{0} \end{array} \right\}$$

$\mathbf{f}(t)$ being a generic failure vector.

Output Observer for *Fault Isolation*

$$\left.\begin{aligned}\mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{B}\mathbf{f}_u(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{f}_y(t)\end{aligned}\right\}$$

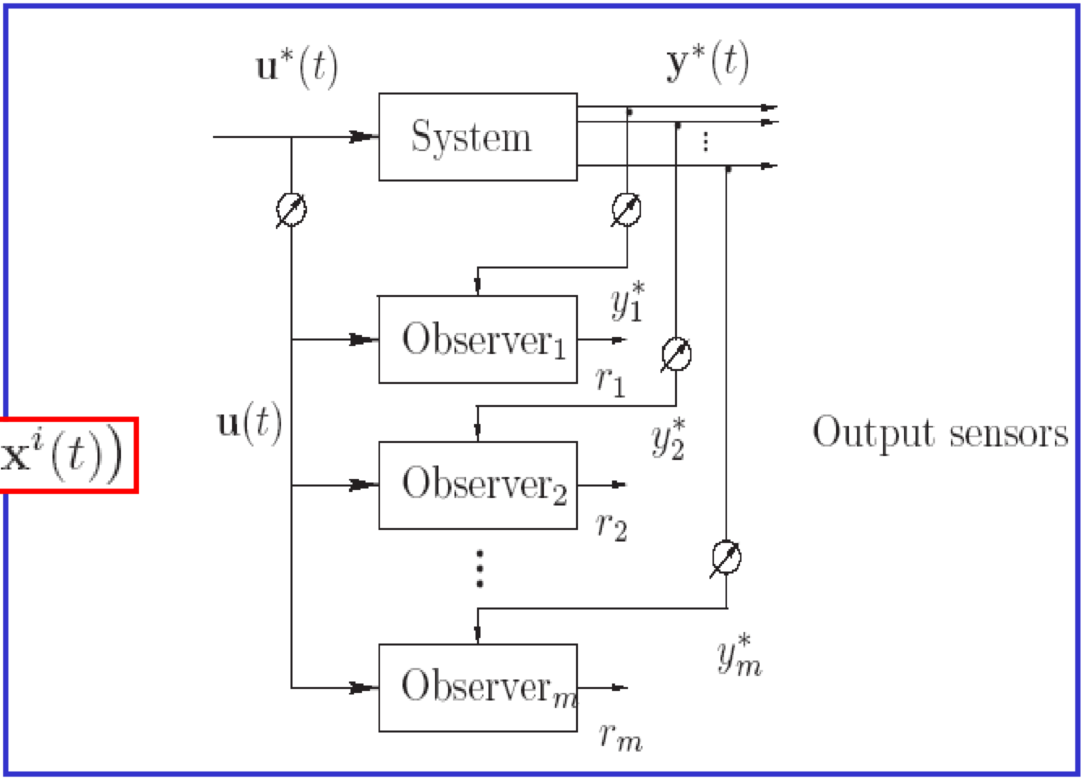
Process model

$$\mathbf{x}^i(t+1) = \mathbf{A}_i\mathbf{x}^i(t) + \mathbf{B}_i\mathbf{u}(t) + \\ + \mathbf{K}_i\left(y_i(t) - \mathbf{C}_i\mathbf{x}^i(t)\right)$$

$$\boxed{r_i(t) = y_i^*(t) - \hat{y}_i(t) = \mathbf{C}_i\left(\mathbf{x}_i(t) - \mathbf{x}^i(t)\right)}$$

$$\boxed{y_i(t) = y_i^*(t) + f(t)}$$

Bank of output observers



Output Observer for *Fault Isolation* (Cont'd)

Bank of output observers

$$\mathbf{x}^i(t+1) = \mathbf{A}_i\mathbf{x}^i(t) + \mathbf{B}_i\mathbf{u}(t) + \mathbf{K}_i\left(y_i(t) - \mathbf{C}_i\mathbf{x}^i(t)\right)$$

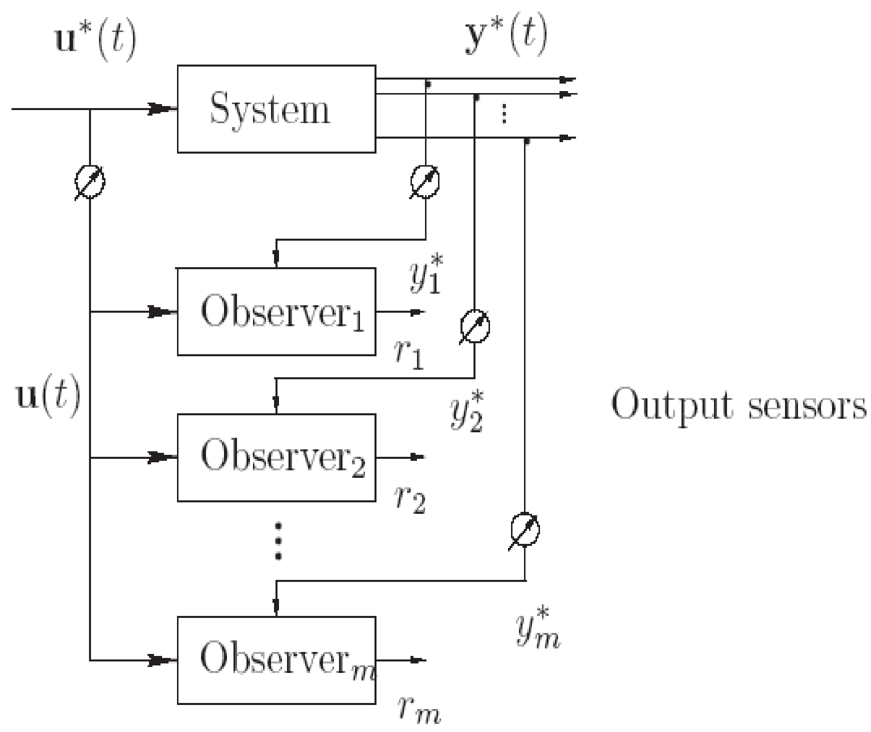
$$\boxed{r_i(t) = y_i^*(t) - \hat{y}_i(t) = \mathbf{C}_i\left(\mathbf{x}_i(t) - \mathbf{x}^i(t)\right)}$$

Fault-free case:

$$\boxed{\lim_{t\to\infty} r_i(t) = \lim_{t\to\infty}\left(y_i(t) - C^i\mathbf{x}^i(t)\right) = 0}$$

Faulty case

$$\boxed{y_i(t) = y_i^*(t) + f(t)}$$



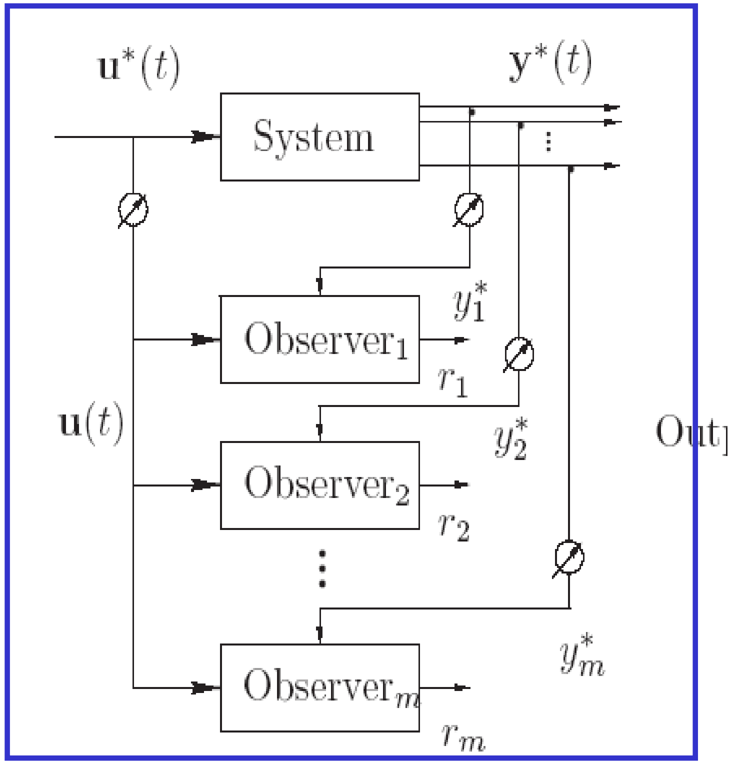
Output Observer for *Fault Isolation* (Cont'd)

Table 4.1: Fault signatures.

	u_1	u_2	$\dots$	u_r	y_1	y_2	$\dots$	y_m
r_{O_1}	1	1	$\dots$	1	1	0	$\dots$	0
r_{O_2}	1	1	$\dots$	1	0	1	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
r_{O_m}	1	1	$\dots$	1	0	0	$\dots$	1

$$\mathbf{x}^i(t+1) = \mathbf{A}_i\mathbf{x}^i(t) + \mathbf{B}_i\mathbf{u}(t) + \mathbf{K}_i\left(y_i(t) - \mathbf{C}_i\mathbf{x}^i(t)\right)$$

Bank of output observers



Fault-free case:

$$\lim_{t\to\infty} r_i(t) = \lim_{t\to\infty}\left(y_i(t) - C^i\mathbf{x}^i(t)\right) = 0$$

Faulty case

$$y_i(t) = y_i^*(t) + f(t)$$

$$\lim_{t\to\infty} r_i(t) \neq 0$$

Unknow Input Observer (UIO)

Disturbance Distribution Matrix (Known)

$$\left.\begin{aligned} \mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}\mathbf{d}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned}\right\}$$

System with disturbance
"Unknown Input"

Unknown Input

Definition: An observer is defined as an *Unknown Input Observer* for the system with disturbance (above), if its state estimation error vector $e_x(t)$ approaches zero asymptotically, regardless of the presence of the unknown input term in the system.

UIO Model

Given:

$$\left.\begin{array}{lll}\mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{E}\mathbf{d}(t) \\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t)\end{array}\right\}$$

The full–order UIO has the following mathematical form

$$\left.\begin{array}{lll}\mathbf{z}(t+1) & = & \mathbf{F}\mathbf{z}(t)+\mathbf{T}\mathbf{B}\mathbf{u}(t)+\mathbf{K}\mathbf{y}(t) \\ \hat{\mathbf{x}}(t) & = & \mathbf{z}(t)+\mathbf{H}\mathbf{y}(t)\end{array}\right\}$$

where $\mathbf{z}(t) \in \Re^n$ is the state of the UIO, $\hat{\mathbf{x}}(t)$ the estimated state vector $\mathbf{x}(t)$, whilst $\mathbf{F}$, $\mathbf{T}$, $\mathbf{H}$ and $\mathbf{K}$ are matrices to be designed to achieve the unknown input de–coupling .

UIO Structure

Plant Model

$$\left.\begin{array}{lcl} \mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{E}\mathbf{d}(t) \\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t) \end{array}\right\}$$

UIO Model

$$\left.\begin{array}{lcl} \mathbf{z}(t+1) & = & \mathbf{F}\mathbf{z}(t)+\mathbf{TB}\mathbf{u}(t)+\mathbf{K}\mathbf{y}(t) \\ \hat{\mathbf{x}}(t) & = & \mathbf{z}(t)+\mathbf{H}\mathbf{y}(t) \end{array}\right\}$$

Observer Design???

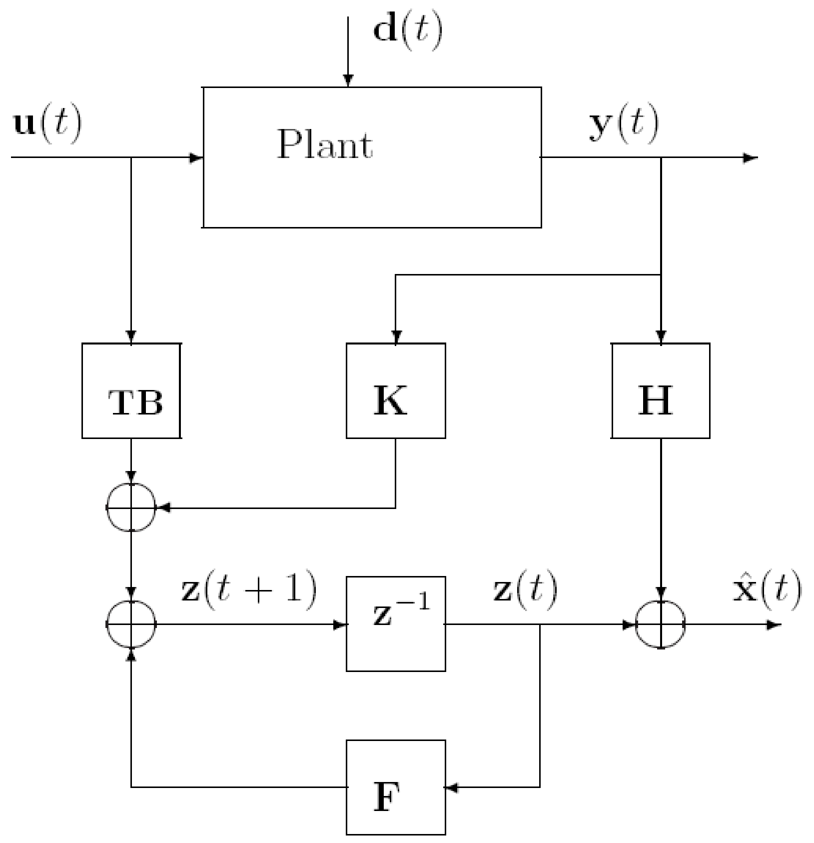


Figure 4.3: The UIO structure.

UIO Design

$$\left.\begin{aligned}\mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}\mathbf{d}(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned}\right\}$$

Plant Model

$$\left.\begin{aligned}\mathbf{z}(t+1) &= \mathbf{F}\mathbf{z}(t) + \mathbf{T}\mathbf{B}\mathbf{u}(t) + \mathbf{K}\mathbf{y}(t)\\ \hat{\mathbf{x}}(t) &= \mathbf{z}(t) + \mathbf{H}\mathbf{y}(t)\end{aligned}\right\}$$

UIO Model

$$e_x(t+1) = x(t+1) - \hat{x}(t+1)$$

State estimation error

$$\boxed{\begin{aligned}\mathbf{e}_x(t+1) &= [\mathbf{A} - \mathbf{HCA} - \mathbf{K}_1\mathbf{C}]\mathbf{e}_x(t) + \big[\mathbf{F} - (\mathbf{A} - \mathbf{HCA} - \mathbf{K}_1\mathbf{C})\big]\mathbf{z}(t)\\ &+ \big[\mathbf{K}_2 - (\mathbf{A} - \mathbf{HCA} - \mathbf{K}_1\mathbf{C})\big]\mathbf{y}(t)\\ &+ \big[\mathbf{T} - (\mathbf{I} - \mathbf{HC})\big]\mathbf{B}\mathbf{u}(t) + (\mathbf{HC} - \mathbf{I})\mathbf{E}\mathbf{d}(t)\end{aligned}}$$

UIO Design (Cont'd)

$$\begin{aligned}\mathbf{e}_x(t+1) &= [\mathbf{A}-\mathbf{HCA}-\mathbf{K}_1\mathbf{C}]\mathbf{e}_x(t)+\big[\mathbf{F}-(\mathbf{A}-\mathbf{HCA}-\mathbf{K}_1\mathbf{C})\big]\mathbf{z}(t)\\ &+ \big[\mathbf{K}_2-(\mathbf{A}-\mathbf{HCA}-\mathbf{K}_1\mathbf{C})\big]\mathbf{y}(t)\\ &+ \big[\mathbf{T}-(\mathbf{I}-\mathbf{HC})\big]\mathbf{Bu}(t)+(\mathbf{HC}-\mathbf{I})\mathbf{Ed}(t)\end{aligned}$$

$$\left.\begin{aligned}\mathbf{z}(t+1) &= \mathbf{Fz}(t)+\mathbf{TBu}(t)+\mathbf{Ky}(t)\\ \hat{\mathbf{x}}(t) &= \mathbf{z}(t)+\mathbf{Hy}(t)\end{aligned}\right\}$$

$$\begin{aligned}(\mathbf{HC}-\mathbf{I})\mathbf{E} &= \mathbf{0}\\ \mathbf{I}-\mathbf{HC} &= \mathbf{T}\\ \mathbf{A}-\mathbf{HCA}-\mathbf{K}_1\mathbf{C} &= \mathbf{F}\\ \mathbf{FH} &= \mathbf{K}_2\end{aligned}$$

the state estimation error will then be:

$$\mathbf{e}_x(t+1)=\mathbf{Fe}_x(t).$$

UIO Design (Cont'd)

$$\left.\begin{array}{lcl} \mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{E}\mathbf{d}(t) \\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t) \end{array}\right\}$$

$$\begin{array}{lcl} (\mathbf{HC}-\mathbf{I})\mathbf{E} & = & \mathbf{0} \\ \mathbf{I}-\mathbf{HC} & = & \mathbf{T} \\ \mathbf{A}-\mathbf{HCA}-\mathbf{K}_1\mathbf{C} & = & \mathbf{F} \\ \mathbf{FH} & = & \mathbf{K}_2 \end{array}$$

$$\left.\begin{array}{lcl} \mathbf{z}(t+1) & = & \mathbf{F}\mathbf{z}(t)+\mathbf{TB}\mathbf{u}(t)+\mathbf{K}\mathbf{y}(t) \\ \hat{\mathbf{x}}(t) & = & \mathbf{z}(t)+\mathbf{H}\mathbf{y}(t) \end{array}\right\}$$

$$\mathbf{e}_x(t+1)=\mathbf{F}\mathbf{e}_x(t).$$

This means that, if all the eigenvalues of **F** are stable, $e_x(t)$ will approach zero asymptotically, *i.e.*, $\hat{x}(t) \to x(t)$. Hence, according to the definition of the UOI described by the model above, it is an UIO for the monitored system. The design of this UIO consists of solving the equation system and making all eigenvalues of the system matrix **F** be stable.

UIO Design (Cont'd)

$$\left.\begin{aligned}\mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{E}\mathbf{d}(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned}\right\} \quad **$$

$$\left.\begin{aligned}\mathbf{z}(t+1) &= \mathbf{F}\mathbf{z}(t)+\mathbf{T}\mathbf{B}\mathbf{u}(t)+\mathbf{K}\mathbf{y}(t)\\ \hat{\mathbf{x}}(t) &= \mathbf{z}(t)+\mathbf{H}\mathbf{y}(t)\end{aligned}\right\} \quad *$$

***

$$\begin{aligned}(\mathbf{H}\mathbf{C}-\mathbf{I})\mathbf{E} &= \mathbf{0}\\ \mathbf{I}-\mathbf{H}\mathbf{C} &= \mathbf{T}\\ \mathbf{A}-\mathbf{H}\mathbf{C}\mathbf{A}-\mathbf{K}_1\mathbf{C} &= \mathbf{F}\\ \mathbf{F}\mathbf{H} &= \mathbf{K}_2\end{aligned}$$

Theorem *Necessary and sufficient conditions for the existence of an UIO* * *for the system defined by Equation* ** *are [Chen & Patton, 1999]:*

1. *rank*$(\mathbf{CE}) =$ *rank*$(\mathbf{E})$,
2. $(\mathbf{A}_1, \mathbf{C})$ *is a detectable pair,*

where $\mathbf{A}_1 = \mathbf{A} - \mathbf{E}(\mathbf{CE})^+\mathbf{CA}$.

A special solution for the matrix $\mathbf{H}$ in conditions *** is given by [Chen & Patton, 1999]:

$$\mathbf{H}^* = \mathbf{E}(\mathbf{CE})^+$$

where $(\cdot)^+$ is the pseudoinverse of the matrix $\mathbf{CE}$.

UIO for Fault (Detection) + Isolation

$$\left.\begin{array}{rcl}\mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{E}\mathbf{d}(t)\\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t)\end{array}\right\}$$

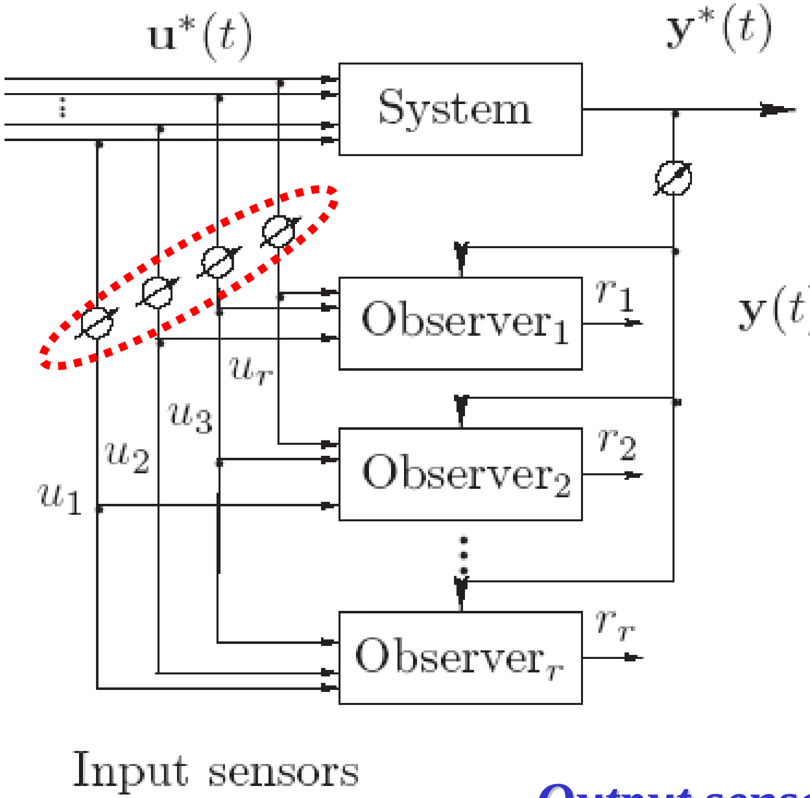
General system

$$\left.\begin{array}{rcl}\mathbf{z}(t+1) & = & \mathbf{F}\mathbf{z}(t)+\mathbf{T}\mathbf{B}\mathbf{u}(t)+\mathbf{K}\mathbf{y}(t)\\ \hat{\mathbf{x}}(t) & = & \mathbf{z}(t)+\mathbf{H}\mathbf{y}(t)\end{array}\right\}$$

UIO model

$$\left.\begin{array}{rcl}\mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{B}\mathbf{f}_u(t)\\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t)+\mathbf{f}_y(t)\end{array}\right\}$$

Process with input faults

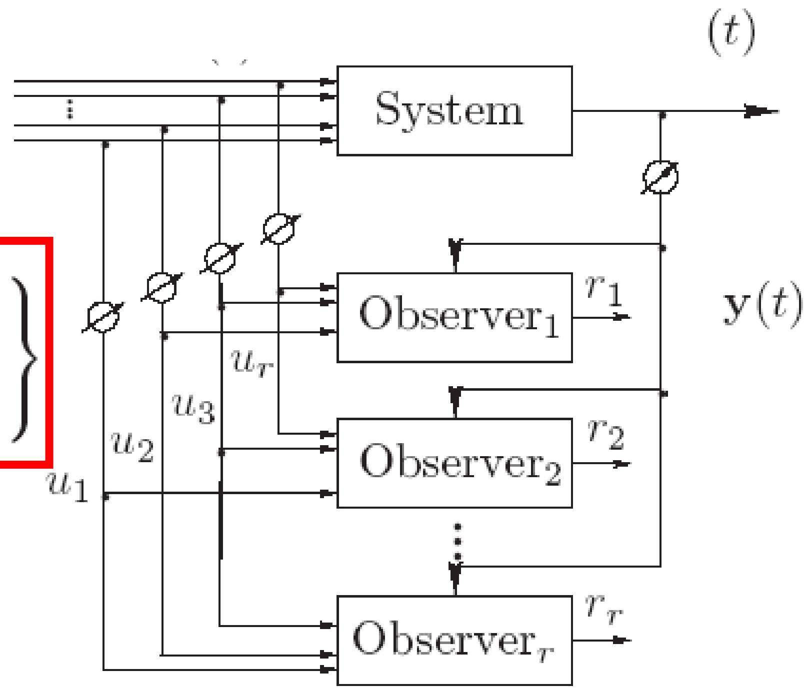


Output sensors are fault-free

UIO for FDI

With reference to the devices for the FDI of the inputs, depicted in Figure 4.5, the structure of the i-th UIO ($i = 1, 2, \ldots, r$) for residual generation [Chen & Patton, 1999], under the assumptions $\tilde{\mathbf{u}}(t) \cong \mathbf{0}$, $\tilde{\mathbf{y}}(t) \cong \mathbf{0}$ and $\mathbf{f}_y(t) = \mathbf{0}$, is the following

$$\left.\begin{aligned} \mathbf{z}^i(t+1) &= (\mathbf{T}^i\mathbf{A} - \mathbf{K}^i\mathbf{C})\mathbf{z}^i(t) + \mathbf{J}^i\mathbf{u}(t) + \mathbf{S}^i\mathbf{y}(t) \\ \mathbf{r}^i(t) &= \mathbf{L}_1^i\mathbf{z}^i(t) + \mathbf{L}_2^i\mathbf{y}(t) \end{aligned}\right\}$$



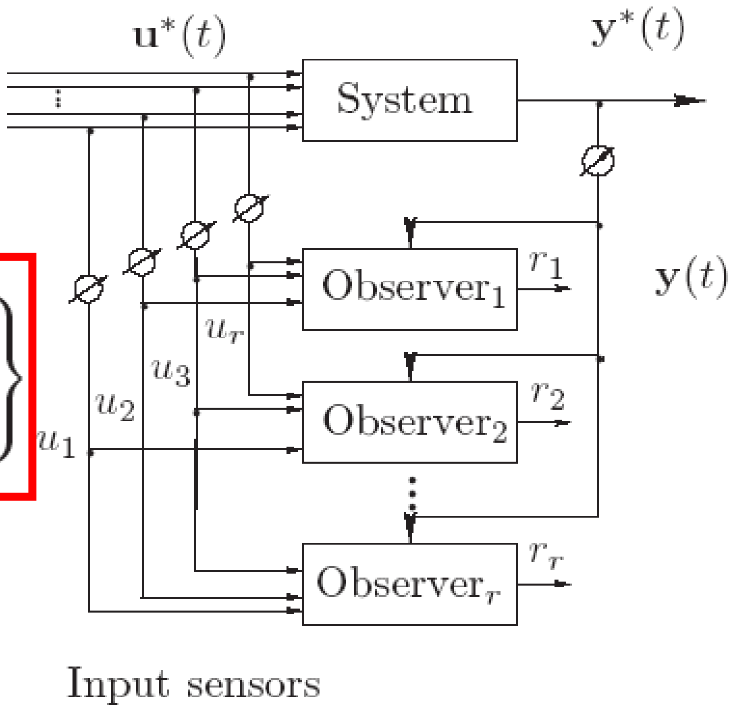
Input sensors

UIO for Residual Generation

UIO for FDI (Cont'd)

UIO for Residual Generation

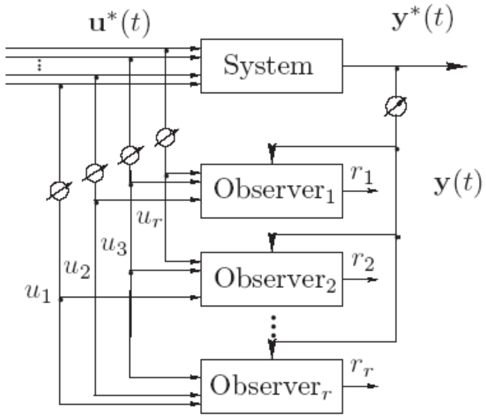
$$\left.\begin{aligned} \mathbf{z}^i(t+1) &= (\mathbf{T}^i\mathbf{A} - \mathbf{K}^i\mathbf{C})\mathbf{z}^i(t) + \mathbf{J}^i\mathbf{u}(t) + \mathbf{S}^i\mathbf{y}(t) \\ \mathbf{r}^i(t) &= \mathbf{L}_1^i\mathbf{z}^i(t) + \mathbf{L}_2^i\mathbf{y}(t) \end{aligned}\right\}$$



where $\mathbf{z}^i(t) \in \Re^n$ denotes the observer state vector, $\mathbf{r}^i(t) \in \Re^m$ is the residual vector and $\mathbf{F}^i$, $\mathbf{J}^i$, $\mathbf{S}^i$, $\mathbf{L}_1^i$ and $\mathbf{L}_2^i$ are matrices to be designed with appropriate dimensions. Let $\mathbf{T}^i$ be a linear transformation of the state $\mathbf{x}(t)$ of the system and define the state estimation error as $\mathbf{e}_x^i(t) = \mathbf{z}^i(t) - \mathbf{T}^i\mathbf{x}(t)$.

UIO for FDI (Cont'd)

$$\left.\begin{array}{lcl} \mathbf{z}^i(t+1) & = & \left(\mathbf{T}^i\mathbf{A} - \mathbf{K}^i\mathbf{C}\right)\mathbf{z}^i(t) + \mathbf{J}^i\mathbf{u}(t) + \mathbf{S}^i\mathbf{y}(t) \\ \mathbf{r}^i(t) & = & \mathbf{L}_1^i\mathbf{z}^i(t) + \mathbf{L}_2^i\mathbf{y}(t) \end{array}\right\}$$



define the state estimation error as $\mathbf{e}_x^i(t) = \mathbf{z}^i(t) - \mathbf{T}^i\mathbf{x}(t)$. On the suppositions $\tilde{\mathbf{u}}(t) = \mathbf{0}$, $\tilde{\mathbf{y}}(t) = \mathbf{0}$, and $\mathbf{f}_y(t) = \mathbf{0}$, it can be shown that the dynamics of the state estimation error become

$$\mathbf{e}_x^i(t+1) = \mathbf{F}^i\mathbf{e}_x^i(t) + \left(\mathbf{F}^i\mathbf{T}^i - \mathbf{T}^i\mathbf{A} + \mathbf{S}^i\mathbf{C}\right)\mathbf{x}(t) + (\mathbf{J}^i - \mathbf{T}^i\mathbf{B})\mathbf{u}(t) - \mathbf{T}^i\mathbf{B}\mathbf{f}_u(t),$$

whilst the residual vector is given by

$$\mathbf{r}^i(t) = \mathbf{L}_1^i\mathbf{e}_x^i(t) + (\mathbf{L}_1^i\mathbf{T}^i + \mathbf{L}_2^i\mathbf{C})\mathbf{x}(t).$$

UIO for FDI (Cont'd)

whilst the residual vector is given by

$$\mathbf{r}^i(t) = \mathbf{L}_1^i \mathbf{e}_x^i(t) + (\mathbf{L}_1^i \mathbf{T}^i + \mathbf{L}_2^i \mathbf{C})\mathbf{x}(t).$$

It can be seen that if

$$\left.\begin{array}{lcl} \mathbf{F}^i\mathbf{T}^i - \mathbf{T}^i\mathbf{A} + \mathbf{S}^i\mathbf{C} & = & \mathbf{0}, \\ \mathbf{J}^i & = & \mathbf{T}^i\mathbf{B}, \\ \mathbf{L}_1^i\mathbf{T}^i + \mathbf{L}_2^i\mathbf{C} & = & \mathbf{0}, \end{array}\right\}$$

UIO for input sensor $\mathbf{f}_u(t)$ fault isolation

$$\boxed{\left.\begin{array}{lcl} \mathbf{e}_x^i(t+1) & = & \mathbf{F}^i\mathbf{e}_x^i(t) + \mathbf{T}^i\mathbf{B}\mathbf{f}_u(t), \\ \mathbf{r}^i(t) & = & \mathbf{L}_1^i\mathbf{e}_i(t). \end{array}\right\}}$$

UIO for FDI (Cont'd)

$$\left.\begin{array}{lcl} \mathbf{z}^i(t+1) & = & (\mathbf{T}^i\mathbf{A} - \mathbf{K}^i\mathbf{C})\mathbf{z}^i(t) + \mathbf{J}^i\mathbf{u}(t) + \mathbf{S}^i\mathbf{y}(t) \\ \mathbf{r}^i(t) & = & \mathbf{L}_1^i\mathbf{z}^i(t) + \mathbf{L}_2^i\mathbf{y}(t) \end{array}\right\}$$

$$\left.\begin{array}{lcl} \mathbf{e}_x^i(t+1) & = & \mathbf{F}^i\mathbf{e}_x^i(t) + \mathbf{T}^i\mathbf{B}\mathbf{f}_u(t), \\ \mathbf{r}^i(t) & = & \mathbf{L}_1^i\mathbf{e}_i(t). \end{array}\right\}$$

If the linear transformation $\mathbf{T}^i$ is chosen as [Chang & Hsu, 1995]

$$* \qquad \mathbf{T}^i = \mathbf{I}_n - \mathbf{B}_i(\mathbf{C}\mathbf{B}_i)^+\mathbf{C} \qquad (4.25)$$

where $\mathbf{B}_i$ is the i-th column of B matrix and $\mathbf{K}^i$ is selected such that $\mathbf{F}^i = \mathbf{T}^i\mathbf{A} - \mathbf{K}^i\mathbf{C}$ is asymptotically stable, then, the solutions of Equation are obtained as

$$** \qquad \left.\begin{array}{lcl} \mathbf{F}^i & = & \mathbf{T}^i\mathbf{A} - \mathbf{K}^i\mathbf{C}, \\ \mathbf{S}^i & = & \mathbf{K}^i + \mathbf{F}^i\mathbf{B}_i(\mathbf{C}\mathbf{B}_i)^+, \\ \mathbf{J}^i & = & \mathbf{T}^i\mathbf{B}, \\ \mathbf{L}_1^i & = & -\mathbf{C}, \\ \mathbf{L}_2^i & = & \left[\mathbf{I}_m - (\mathbf{C}\mathbf{B}_i)(\mathbf{C}\mathbf{B}_i)^+\right]. \end{array}\right\}$$

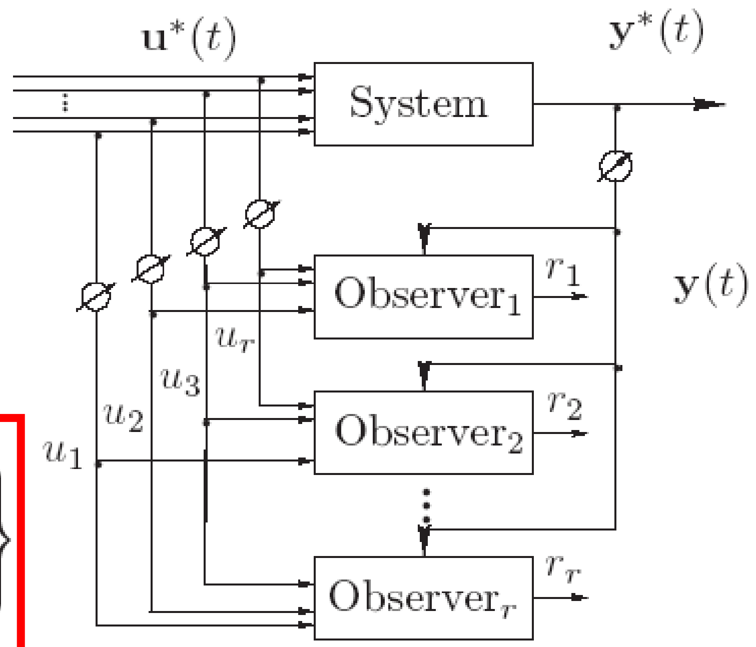
$$\left.\begin{array}{lcl} \mathbf{F}^i\mathbf{T}^i - \mathbf{T}^i\mathbf{A} + \mathbf{S}^i\mathbf{C} & = & \mathbf{0}, \\ \mathbf{J}^i & = & \mathbf{T}^i\mathbf{B}, \\ \mathbf{L}_1^i\mathbf{T}^i + \mathbf{L}_2^i\mathbf{C} & = & \mathbf{0}, \end{array}\right\}$$

The selection of the $\mathbf{B}_i$ matrix in Equations * and ** sets to zero the i-th column of the $\mathbf{J}^i$ matrix. That is, the estimation error and then the residual of the i-th UIO become independent of the i-th system input.

UIO for FDI (Cont'd)

$$\left.\begin{aligned} \mathbf{e}_x^i(t+1) &= \mathbf{F}^i\mathbf{e}_x^i(t) + \mathbf{T}^i\mathbf{B}\mathbf{f}_u(t), \\ \mathbf{r}^i(t) &= \mathbf{L}_1^i\mathbf{e}_i(t). \end{aligned}\right\} \quad *$$

$$\left.\begin{aligned} \mathbf{z}^i(t+1) &= (\mathbf{T}^i\mathbf{A} - \mathbf{K}^i\mathbf{C})\mathbf{z}^i(t) + \mathbf{J}^i\mathbf{u}(t) + \mathbf{S}^i\mathbf{y}(t) \\ \mathbf{r}^i(t) &= \mathbf{L}_1^i\mathbf{z}^i(t) + \mathbf{L}_2^i\mathbf{y}(t) \quad ** \end{aligned}\right\}$$



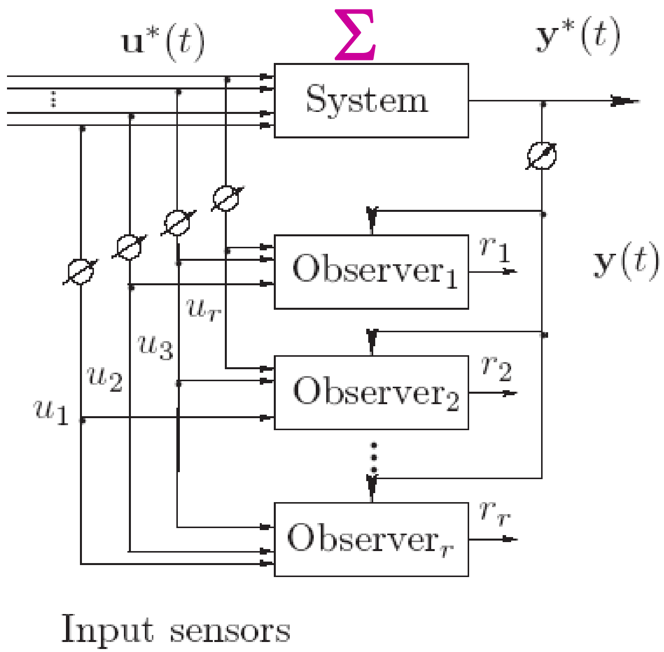
Input sensors

The selection of the $\mathbf{B}_i$ matrix in Equations * and ** sets to zero the i-th column of the $\mathbf{J}^i$ matrix. That is, the estimation error and then the residual of the i-th UIO become independent of the i-th system input.

UIO for FDI (Cont'd)

$$\left.\begin{aligned} \mathbf{e}_x^i(t+1) &= \mathbf{F}^i \mathbf{e}_x^i(t) + \mathbf{T}^i \mathbf{B} \mathbf{f}_u(t), \\ \mathbf{r}^i(t) &= \mathbf{L}_1^i \mathbf{e}_i(t). \end{aligned}\right\}$$

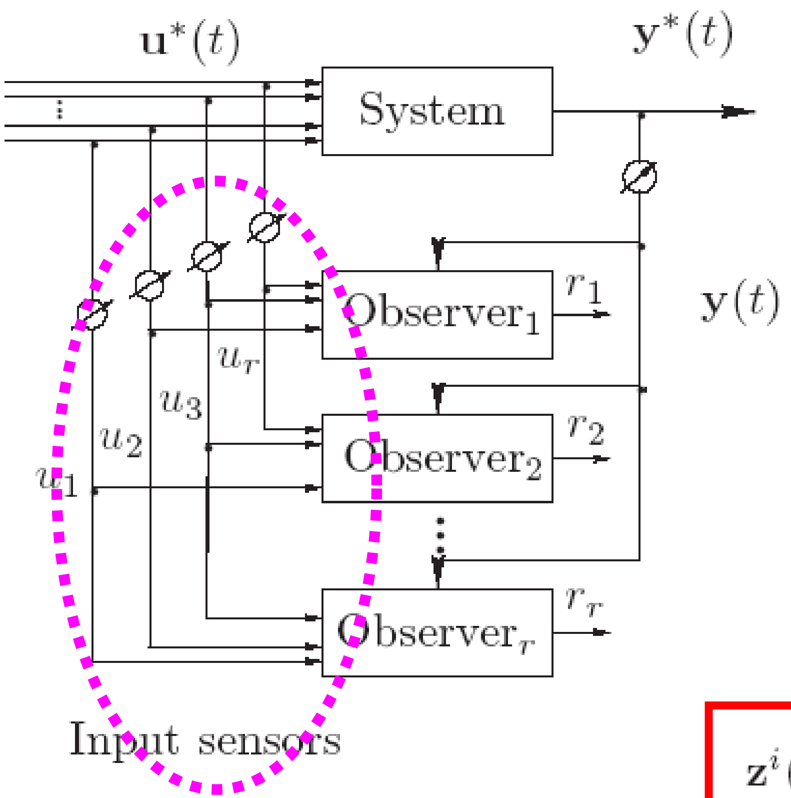
$$\left.\begin{aligned} \mathbf{z}^i(t+1) &= \left(\mathbf{T}^i \mathbf{A} - \mathbf{K}^i \mathbf{C}\right) \mathbf{z}^i(t) + \mathbf{J}^i \mathbf{u}(t) + \mathbf{S}^i \mathbf{y}(t) \\ \mathbf{r}^i(t) &= \mathbf{L}_1^i \mathbf{z}^i(t) + \mathbf{L}_2^i \mathbf{y}(t) \end{aligned}\right\}$$



Under the hypothesis of observability of the system $\underline{\Sigma}$ and in the absence of input faults ($\mathbf{f}_u(t) = \mathbf{0}$), it can be seen that the i–th residual vector reaches zero as t approaches infinity and the rate of convergence depends on the position of the eigenvalues of $\mathbf{T}^i \mathbf{A} - \mathbf{K}^i \mathbf{C}$ matrix inside the unit circle.

In the presence of a fault on the i–th input, the i–th residual reaches zero asymptotically while the residuals of the $r-1$ remaining observers are sensitive to the fault signal. This situation leads to the possibility of unique detection and isolation of all process input faults.

Fault Isolation with UIO



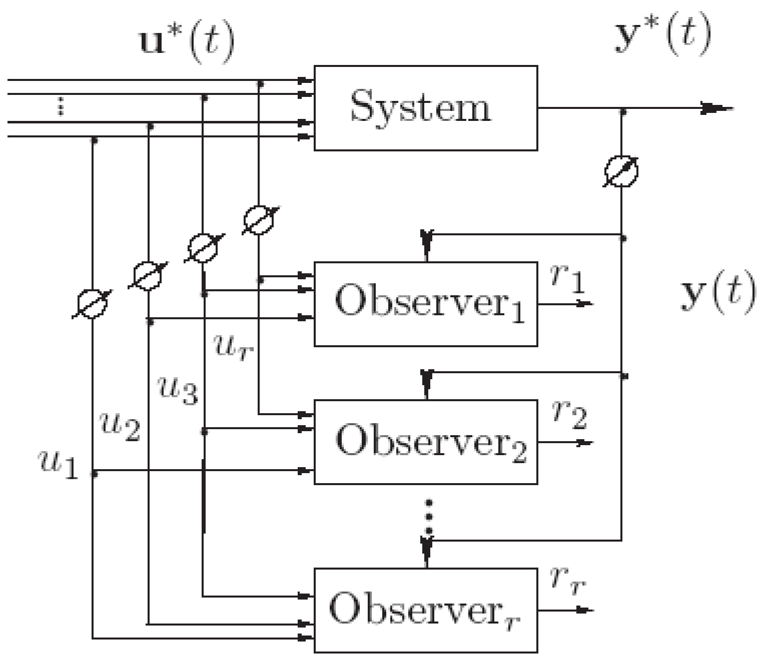
Each observer is insensitive to one input sensor:

Table 4.1: Fault signatures.

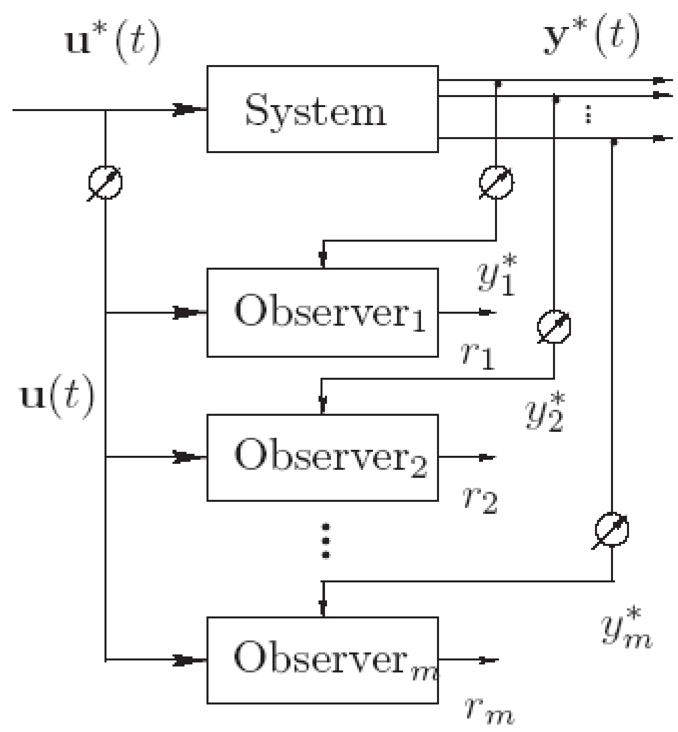
	u_1	u_2	$\dots$	u_r	y_1	y_2	$\dots$	y_m
r_{UIO_1}	0	1	$\dots$	1	1	1	$\dots$	1
r_{UIO_2}	1	0	$\dots$	1	1	1	$\dots$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
r_{UIO_r}	1	1	$\dots$	0	1	1	$\dots$	1

$$\boxed{\left.\begin{aligned} \mathbf{z}^i(t+1) &= (\mathbf{T}^i\mathbf{A} - \mathbf{K}^i\mathbf{C})\mathbf{z}^i(t) + \mathbf{J}^i\mathbf{u}(t) + \mathbf{S}^i\mathbf{y}(t) \\ \mathbf{r}^i(t) &= \mathbf{L}_1^i\mathbf{z}^i(t) + \mathbf{L}_2^i\mathbf{y}(t) \end{aligned}\right\}}$$

Multiple FDI



Input sensor FDI



Input sensor FDI

Multiple FDI (Cont'd)

Table 4.1: Fault signatures.

		u_1	u_2	$\dots$	u_r	y_1	y_2	$\dots$	y_m
UIO	r_{UIO_1}	0	1	$\dots$	1	1	1	$\dots$	1
	r_{UIO_2}	1	0	$\dots$	1	1	1	$\dots$	1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	r_{UIO_r}	1	1	$\dots$	0	1	1	$\dots$	1
Dynamic Observers	r_{O_1}	1	1	$\dots$	1	1	0	$\dots$	0
	r_{O_2}	1	1	$\dots$	1	0	1	$\dots$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	r_{O_m}	1	1	$\dots$	1	0	0	$\dots$	1

Fault Signature

Multiple FDI (Cont'd)

Table 4.1: Fault signatures.

	u_1	u_2	$\dots$	u_r	y_1	y_2	$\dots$	y_m
r_{UIO_1}	0	1	$\dots$	1	1	1	$\dots$	1
r_{UIO_2}	1	0	$\dots$	1	1	1	$\dots$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
r_{UIO_r}	1	1	$\dots$	0	1	1	$\dots$	1
r_{O_1}	1	1	$\dots$	1	1	0	$\dots$	0
r_{O_2}	1	1	$\dots$	1	0	1	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
r_{O_m}	1	1	$\dots$	1	0	0	$\dots$	1

i) The residuals which are affected by the input and output faults are described by an entry '1' in the corresponding table entry, while an entry '0' means that the input or output fault does not affect the corresponding residual.

ii) Note how multiple faults in the system outputs can be isolated since a fault on the i–th output signal affects only the residual function r_{O_i} of the output observer driven by the i–th output, but all the UIO or UIKF residual functions r_{UIO_i}. On the other hand, multiple faults on the inputs cannot be isolated by means of this technique since all the residual functions are sensitive to faults regarding different inputs.

Residual Disturbance *Robustness*

- Residuals decoupled from disturbance
- Robust residual generator
- Disturbance effect minimisation
- Measurement errors

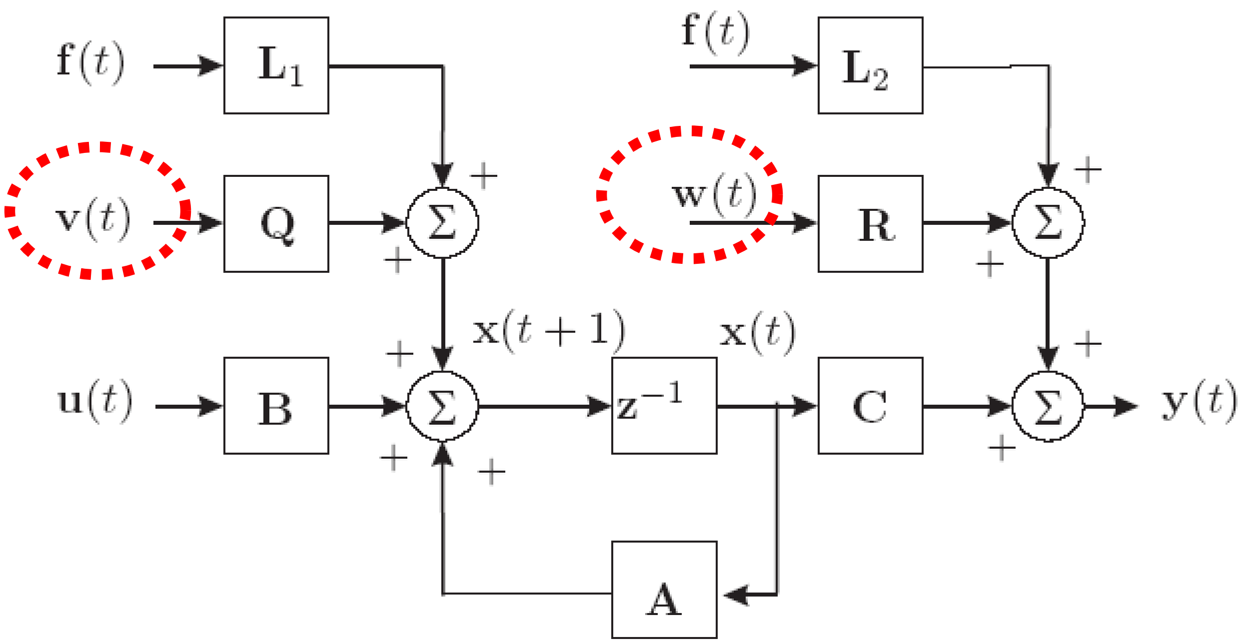


Figure 2.14: MIMO process with faults and noises.

FDI with *Noisy Measurements*

$$\begin{cases} \mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{Q}\mathbf{v}(t) + \mathbf{L}_1\mathbf{f}(t) \\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t) + \mathbf{R}\mathbf{w}(t) + \mathbf{L}_2\mathbf{f}(t) \end{cases}$$

✓ Model with fault and noise

$$\boxed{\begin{cases} \mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{Q}\mathbf{v}(t) \\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t) + \mathbf{R}\mathbf{w}(t) \end{cases}}$$

➢ Model with noise only: Kalman filter!

Kalman Filter Design

$$
* \quad \begin{cases} \mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{Q}\mathbf{v}(t) \\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t) + \mathbf{R}\mathbf{w}(t) \end{cases}
$$

With reference to the time–invariant, discrete–time, linear dynamic system described by Equation * the i-th KF for the i-th output has the structure [Jazwinski, 1970]:

$$
\boxed{\begin{aligned} \mathbf{x}_F^i(t+1|t) &= \mathbf{A}\mathbf{x}_F^i(t|t) + \mathbf{B}\mathbf{u}(t) \\ y_F^i(t+1|t) &= \mathbf{C}_i\mathbf{x}_F^i(t+1|t) \end{aligned}}
$$

(State and Output Prediction)

Kalman Filter Design (Cont'd)

$$\mathbf{P}(t+1|t) = \mathbf{A}\mathbf{P}(t|t)\mathbf{A}^T + \mathbf{Q}$$

$$\mathbf{K}_i(t+1) = \mathbf{P}(t+1|t)\mathbf{C}_i^T\left[\mathbf{C}_i\mathbf{P}(t+1|t)\mathbf{C}_i^T + \mathbf{R}\right]^{-1}$$

$$\mathbf{x}_F^i(t+1|t+1) = \mathbf{x}_F^i(t+1|t) + \mathbf{K}_i(t+1)\left[y_i(t+1) - \hat{y}_F^i(t+1|t)\right]$$

$$\begin{aligned}\mathbf{P}(t+1|t+1) = &\left[\mathbf{I} - \mathbf{K}_i(t+1)\mathbf{C}_i\right]\mathbf{P}(t+1|t)\left[\mathbf{I} - \mathbf{K}_i(t+1)\mathbf{C}_i\right]^T + \\ &+\mathbf{K}_i(t+1)\mathbf{R}\mathbf{K}_i^T(t+1).\end{aligned}$$

(Vector Updates)

The variables $\mathbf{x}_F^i(t+1|t)$ and $y_F^i(t+1|t)$ are the one step prediction of the state and of the output of the process, respectively. $\mathbf{x}_F^i(t|t)$ is the state estimation given by the filter, $\mathbf{C}_i$ the i-th row of the output distribution matrix $\mathbf{C}$, $\mathbf{P}(t+1|t)$ is the covariance matrix of the one step prediction error $\mathbf{x}(t+1) - \mathbf{x}_F^i(t+1|t)$ whilst $P(t|t)$ is the covariance matrix of the filtered state error $\mathbf{x}(t) - \mathbf{x}_F^i(t|t)$.

Kalman Filter Design (Cont'd)

$$\mathbf{P}(t+1|t) = \mathbf{A}\mathbf{P}(t|t)\mathbf{A}^T + \mathbf{Q}$$

$$\mathbf{K}_i(t+1) = \mathbf{P}(t+1|t)\mathbf{C}_i^T\left[\mathbf{C}_i\mathbf{P}(t+1|t)\mathbf{C}_i^T + \mathbf{R}\right]^{-1}$$

$$\mathbf{x}_F^i(t+1|t+1) = \mathbf{x}_F^i(t+1|t) + \mathbf{K}_i(t+1)\left[y_i(t+1) - \hat{y}_F^i(t+1|t)\right]$$

$$\begin{aligned}\mathbf{P}(t+1|t+1) = \; & \left[\mathbf{I} - \mathbf{K}_i(t+1)\mathbf{C}_i\right]\mathbf{P}(t+1|t)\left[\mathbf{I} - \mathbf{K}_i(t+1)\mathbf{C}_i\right]^T + \\ & +\mathbf{K}_i(t+1)\mathbf{R}\mathbf{K}_i^T(t+1).\end{aligned}$$

(Vector Updates)

$\mathbf{Q}$ is the covariance matrix of the input vector noise $\tilde{\mathbf{u}}(t)$ and $\mathbf{R}$ is the variance of the i-th component of the output noise $\tilde{\mathbf{y}}(t)$. $\mathbf{K}_i(t+1)$ is the time-variant gain of the filter and $y_i(t)$ is the i-th component of the measured output $\mathbf{y}(t)$.

$$\begin{cases} \mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{Q}\mathbf{v}(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{R}\mathbf{w}(t) \end{cases}$$

Kalman Filter Properties (Cont'd)

- Defined: $e(t) = x(t) - \hat{x}(t)$
  - It minimises: $\mathrm{E}\left[e^T(t)\, e(t)\right] \equiv P(t)$
  - *i.e.* the mean square error & the error covariance matrix
- Filter gain $K(t)$
  - Solution of the difference equation:

$$\boxed{\begin{aligned} P(t+1) = {} & -A\,P(t)\,C^T\left[R + C\,P(t)\,C^T\right]^{-1} C\,P(t)\,A^T + \\ & + A\,P(t)\,A + Q \end{aligned}}$$

Kalman Filtering for FDI

$$* \quad \begin{cases} \mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{Q}\mathbf{v}(t) + \mathbf{L}_1\mathbf{f}(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{R}\mathbf{w}(t) + \mathbf{L}_2\mathbf{f}(t) \end{cases}$$

1) It can be proved that the innovation $e_i(t+1) = y_i(t+1) - y_F^i(t+1|t) = y_i(t+1) - \mathbf{C}_i\mathbf{x}_F^i(t+1|t)$ is a zero-mean white process when all the assumptions regarding the system * and the statistical characteristics of the noises described by Equation * are completely fulfilled. A Riccati equation is ob-

2) In the presence of a fault on the i-th output ($f_{y_i}(t) \neq 0$), the stochastic properties (mean–value, variance and whiteness, etc) of the innovation process $e_i(t)$ change abruptly so that the fault detection can be based on these variations [Basseville, 1988].

3) Finally, note how multiple faults in outputs can be isolated since a fault on the i-th output affects only the innovation of the KF driven by the i-th output and all the innovation of the filters with unknown input.

Kalman Filtering for FDI (Cont'd)

$$\begin{cases} \mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{Q}\mathbf{v}(t) + \mathbf{L}_1\mathbf{f}(t) \\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t) + \mathbf{R}\mathbf{w}(t) + \mathbf{L}_2\mathbf{f}(t) \end{cases}$$

$$\begin{cases} \mathbf{x}(t+1) & = & \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{Q}\mathbf{v}(t) \\ \mathbf{y}(t) & = & \mathbf{C}\mathbf{x}(t) + \mathbf{R}\mathbf{w}(t) \end{cases}$$

Innovation

$$\boxed{e_i(t+1) = y_i(t+1) - y_F^i(t+1) = y_i(t+1) - C_i\, x_F^i(t+1|t)}$$

(i) Because of the linear property of the identified model and because of the additive effect of the faults on the system, it may easily be shown that the effect of the change on the innovation is also additive.

(ii) Any abrupt change in measurements due to a fault is reflected in a change in the mean value and in the standard deviation of innovations.

Kalman Filtering for FDI (Cont'd)

$$\begin{cases} \mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{Q}\mathbf{v}(t) + \mathbf{L}_1\mathbf{f}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{R}\mathbf{w}(t) + \mathbf{L}_2\mathbf{f}(t) \end{cases}$$

$$\begin{cases} \mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{Q}\mathbf{v}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{R}\mathbf{w}(t) \end{cases}$$

Innovation

$$\boxed{e_i(t+1) = y_i(t+1) - y_F^i(t+1) = y_i(t+1) - C_i\, x_F^i(t+1 \mid t)}$$

In particular, since the KF produces zero–mean and independent white residuals with the system in normal operation, a method for FDI consists of testing how much the sequence of innovations has deviated from the white noise hypothesis.

KF Residual Evaluation…

Which thresholds???

$$\begin{cases} J\left(\mathbf{r}(t)\right) \leq \varepsilon(t) & \text{for} \quad \mathbf{f}(t) = \mathbf{0} \\ J\left(\mathbf{r}(t)\right) > \varepsilon(t) & \text{for} \quad \mathbf{f}(t) \neq \mathbf{0} \end{cases}$$

Innovation

$$\boxed{e_i(t+1) = y_i(t+1) - y_F^i(t+1) = y_i(t+1) - C_i\, x_F^i(t+1\,|\,t)}$$

Kalman Filtering for FDI (Cont'd)

$$\boxed{r(t) \equiv e_i(t+1) = y_i(t+1) - y_F^i(t+1) = y_i(t+1) - C_i\, x_F^i(t+1\,|\,t)}$$

Innovation or Residual r(t)

(i) Statistical Tests

Zero-mean

$$\overline{r}(t) = E[r(t)] = \frac{1}{t}\sum_{j=1}^{t} r(j)$$

&

variance

$$\sigma_r^2(t) = E[r^2(t)] = \frac{1}{t}\sum_{j=1}^{t} r^2(j)$$

Kalman Filtering for FDI (Cont'd)

$$\boxed{r(t) \equiv e_i(t+1) = y_i(t+1) - y_F^i(t+1) = y_i(t+1) - C_i\, x_F^i(t+1\,|\,t)}$$

Innovation or Residual r(t)

(ii) Statistical Tests

Whiteness test

$$R_\tau^t(\tau) = \frac{1}{t}\sum_{j=1}^{t} r(j)r(j+\tau),$$

χ^2 – type

$$\zeta_\tau^M(t) = \frac{t}{R_\tau^t(0)^2}\sum_{\tau=1}^{M}\left(R_\tau^t(\tau)\right)^2$$

which are computed in a growing window. The parameter $\zeta_\tau^M(t)$ is a chi–squared random variable with M degrees of freedom.

Kalman Filtering for FDI (Cont'd)

Whiteness test

$$R_r^t(\tau) = \frac{1}{t}\sum_{j=1}^{t} r(j)r(j+\tau),$$

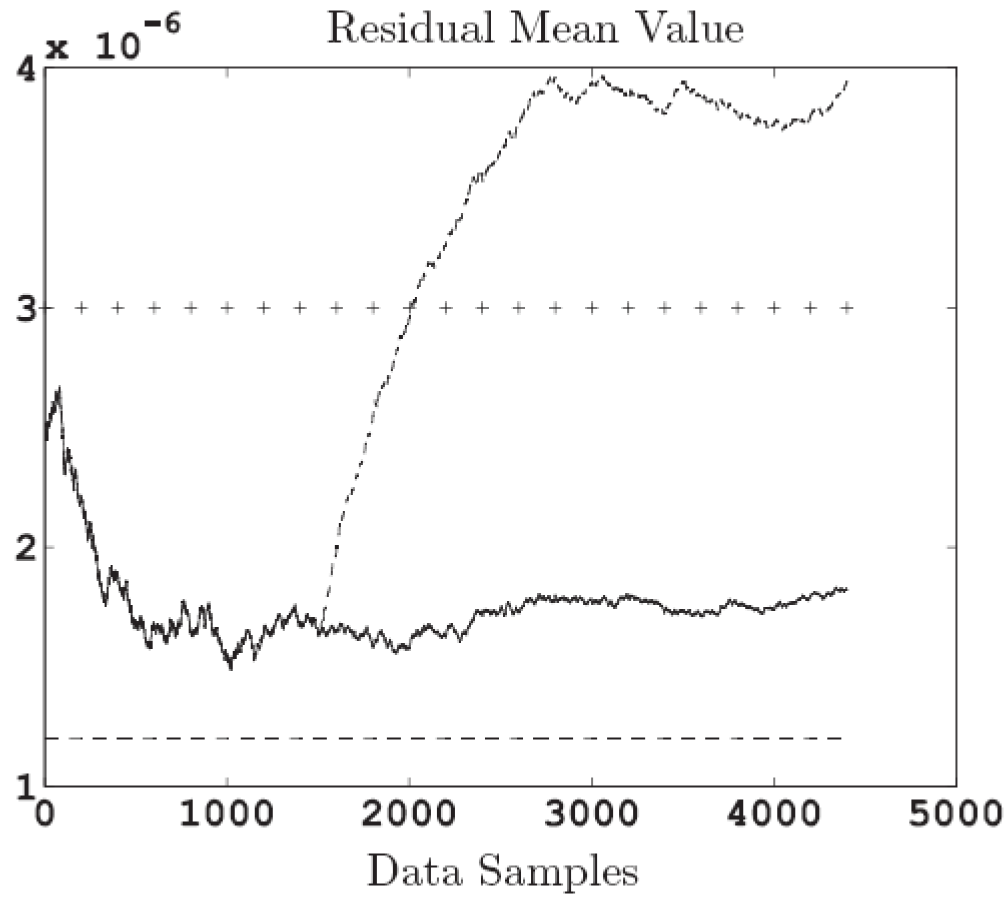
χ^2 – type

$$\zeta_r^M(t) = \frac{t}{R_r^t(0)^2}\sum_{\tau=1}^{M}\left(R_r^t(\tau)\right)^2$$

If a system abnormality occurs, the statistics of $r(t)$ change, so the comparison of $\overline{r}(t)$ and $\zeta_r^M(t)$ with a threshold ϵ fixed under no faults conditions, becomes the detection rule 2.17. In particular, such a threshold can be settled as previously seen or, with the aid of chi–squared tables, $\epsilon = \chi_\beta^2(M)$ can be computed as a function of the false–alarms probability β and of the window size M.

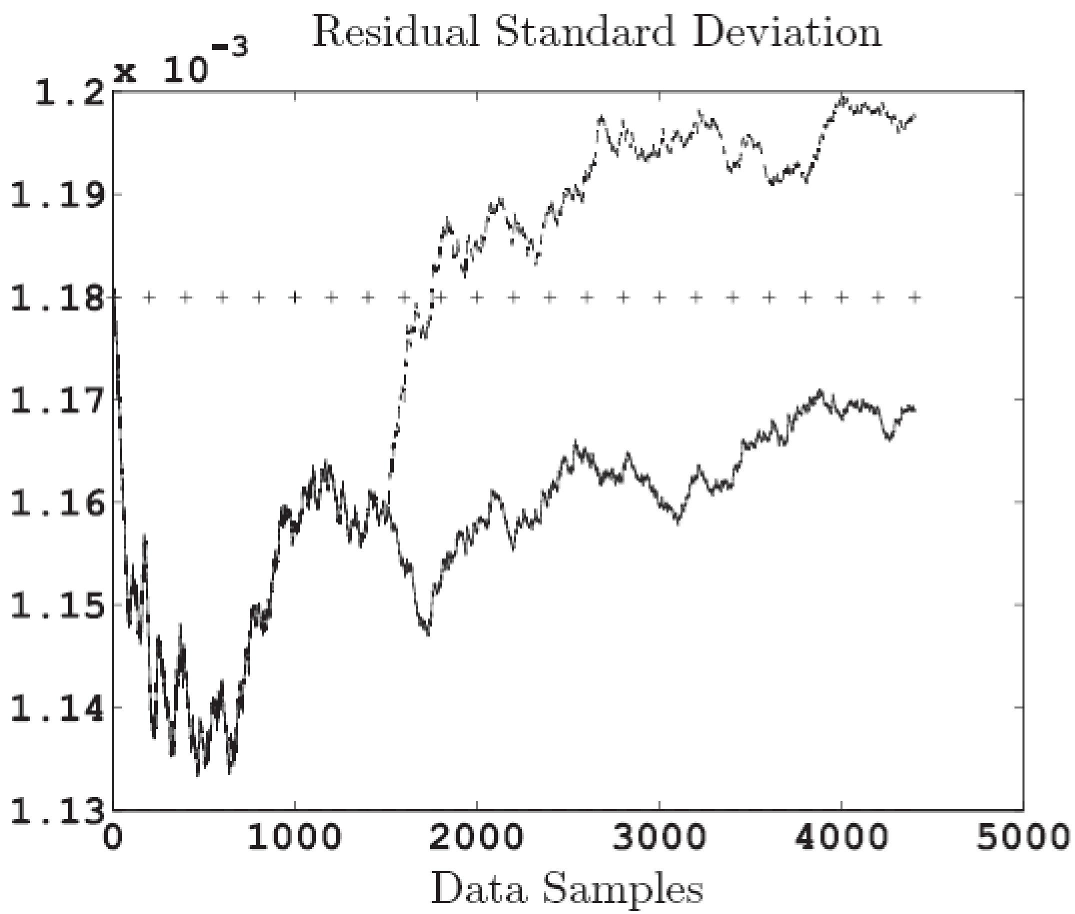
KF Residuals: Mean-value (example)

Fault-free
&
faulty
residuals



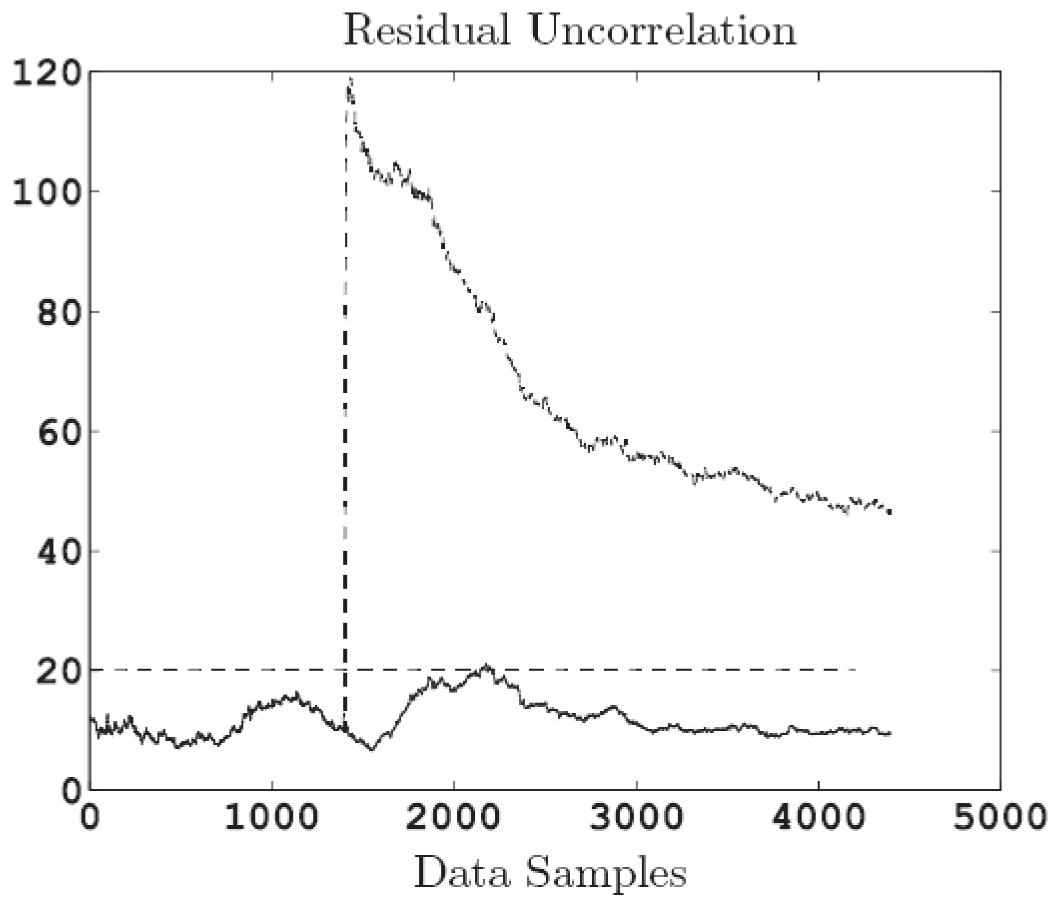
KF Residuals: Standard deviation (example)

Fault-free & faulty residuals



KF Residuals: Whiteness test (example)

Fault-free & faulty residuals



Conclusion

- ✓ Model-Based FDI
- ✓ Analytical Redundancy
- ✓ State-Space Models
- ✓ Residual Generation
  - ✓ Unknown Input Observers UIO
  - ✓ Dynamic Observers / Kalman Filters
- ✓ Residual Evaluation/Change Detection