

# Dynamic Model Identification and Model-Based Design of Residual Generators for the FDI of the MEX System

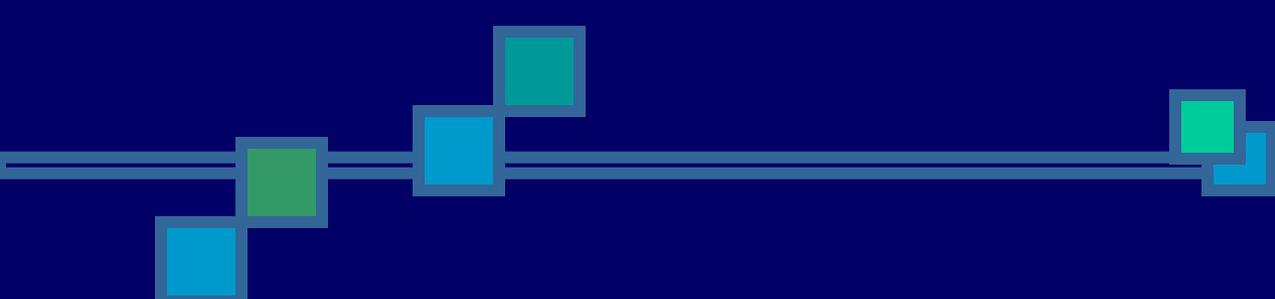
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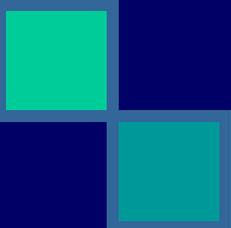
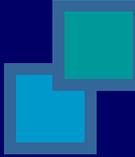
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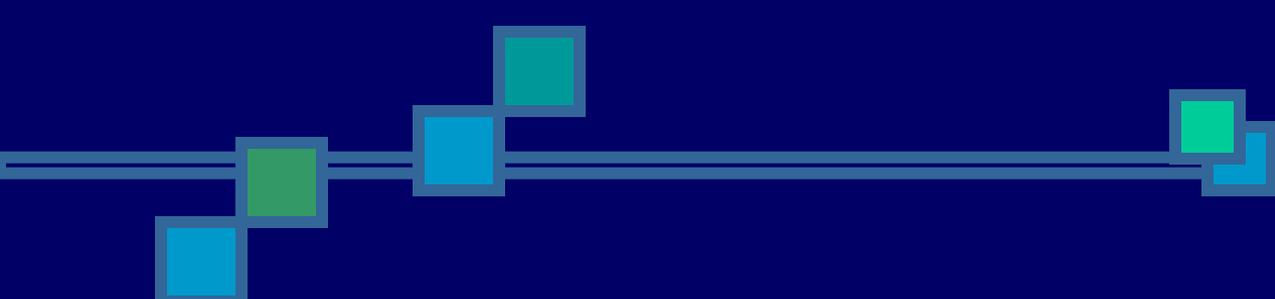
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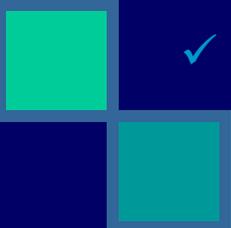
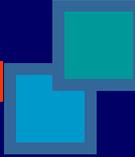


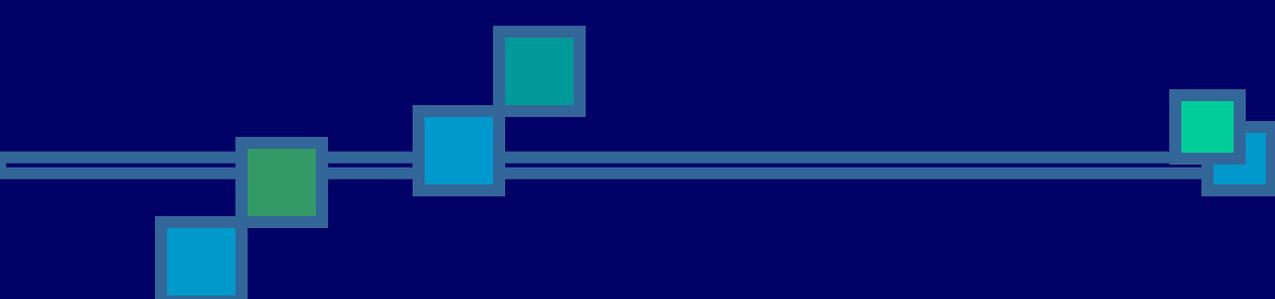
# Introduction

- 
- ❖ Motivation for Robust Identification and FDI Design
  - ❖ Residual Filter Design General Description
  - ❖ Robustness Issue Overview
  - ❖ Fault Model Description
  - ❖ Residual Evaluation Technique
- 

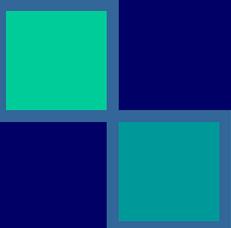


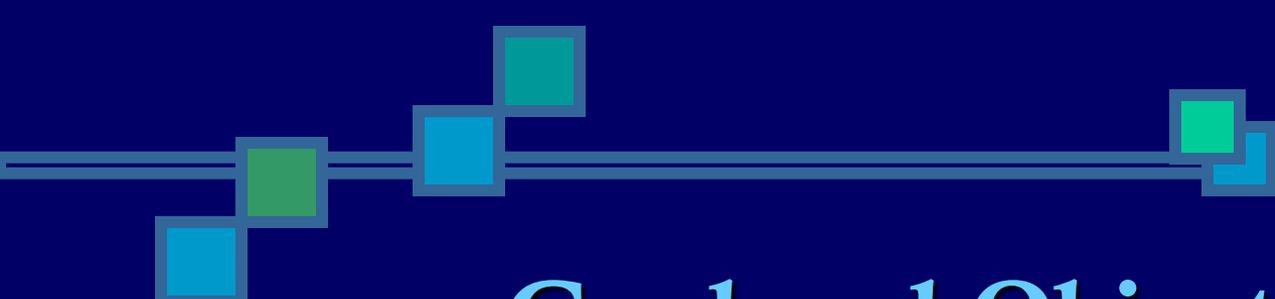
# Vision Statement (i)

- 
- ✓ The *Method* is a model-based FDI scheme relying on:
    - the identification of a linear model of the monitored process.
    - the use of dynamic observers or filters for the residual generation task.
  - ✓ The key point consists of the integration of the dynamic system identification with the design of the residual generation system for diagnostic purpose.
- 



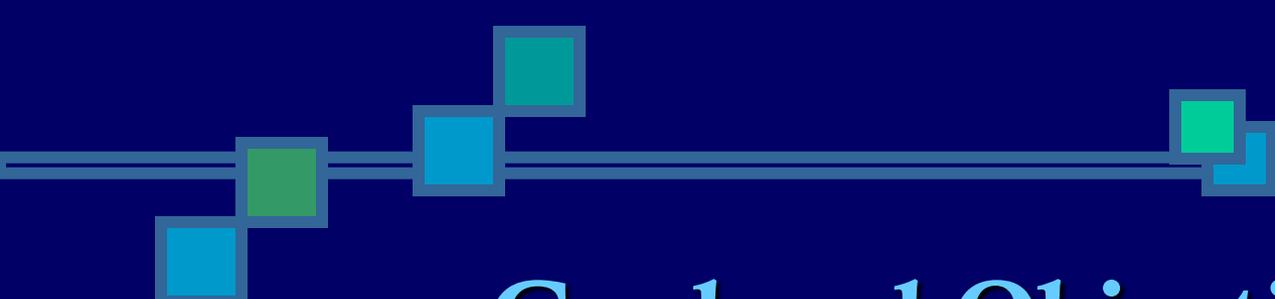
## Vision Statement (ii)

- 
- ✓ *Great care has been given to:*
    - *the robustness of the processing architecture solution.*
    - *the evaluation of the performance that can be achieved.*
  - ✓ *This particular FDI technique has been applied successfully in studies involving also navigation systems of Unmanned Aerial Vehicles.*
- 

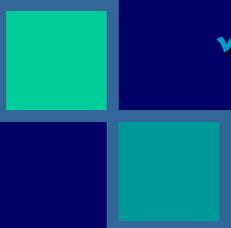


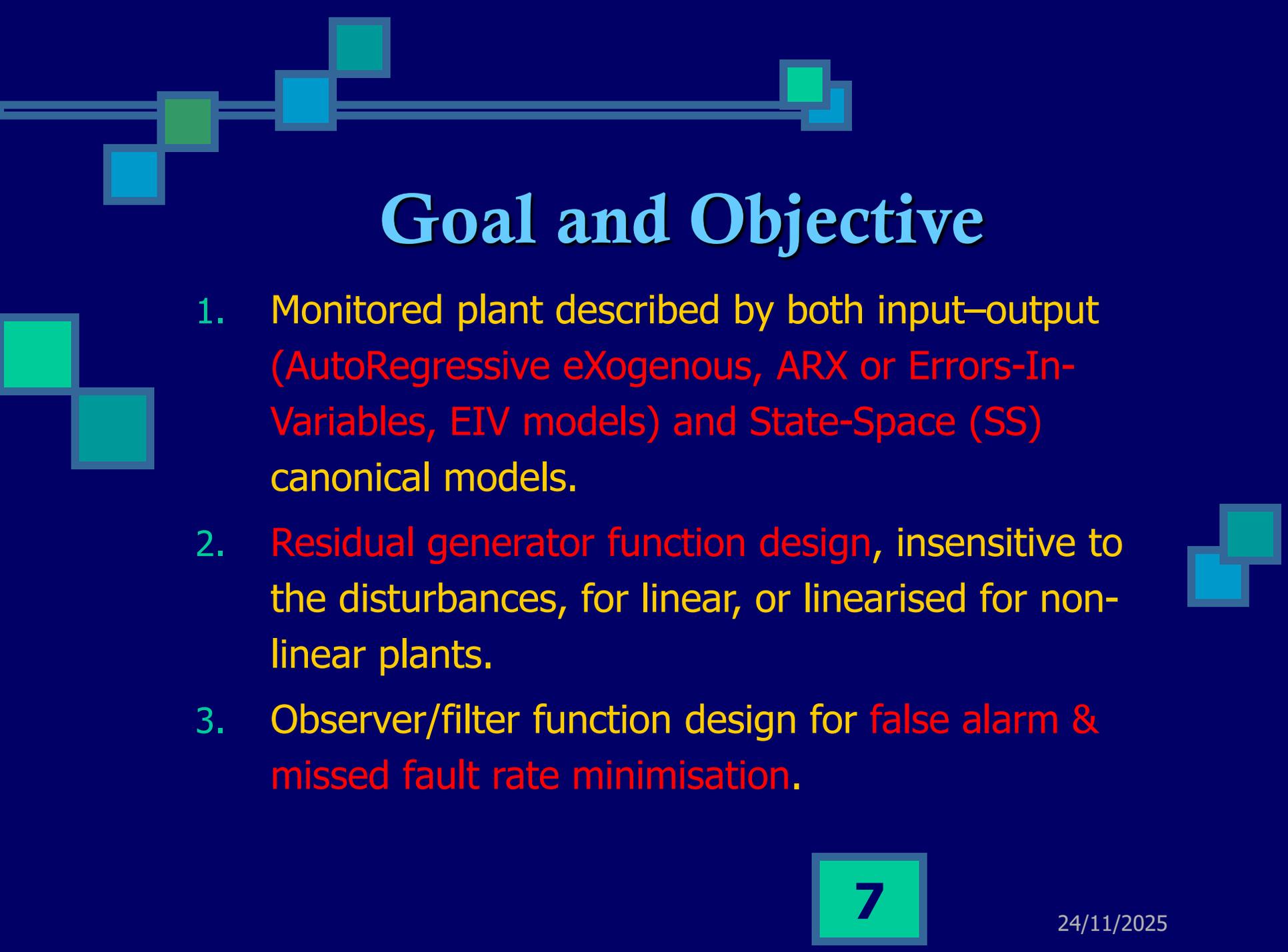
## Goal and Objective (i)

- ✓ Dynamic identification in the discrete-time domain in connection with the design and the use of observers or filters for the *robust* residual generation.
  - ✓ Emphasis on the *robustness versus modelling errors* (*i.e.* the model-reality mismatch), nonlinearity and noise signals affecting the data acquired from the system.
- 



## Goal and Objective (ii)

- 
- ✓ Design of dynamic **observers or filters** exploited for the detection & isolation of faults affecting the process under investigation.
  - ✓ Computation of **fixed thresholds** for performing the residual evaluation stage, based on a geometric analysis of the residual signals.
    - Thresholds have been settled to achieve robust performance of the FDI scheme.
- 

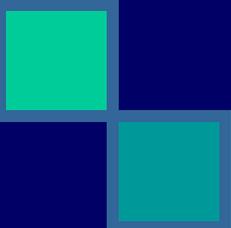


# Goal and Objective

1. Monitored plant described by both input–output (AutoRegressive eXogenous, ARX or Errors-In-Variables, EIV models) and State-Space (SS) canonical models.
2. Residual generator function design, insensitive to the disturbances, for linear, or linearised for non-linear plants.
3. Observer/filter function design for false alarm & missed fault rate minimisation.



# Today's Situation

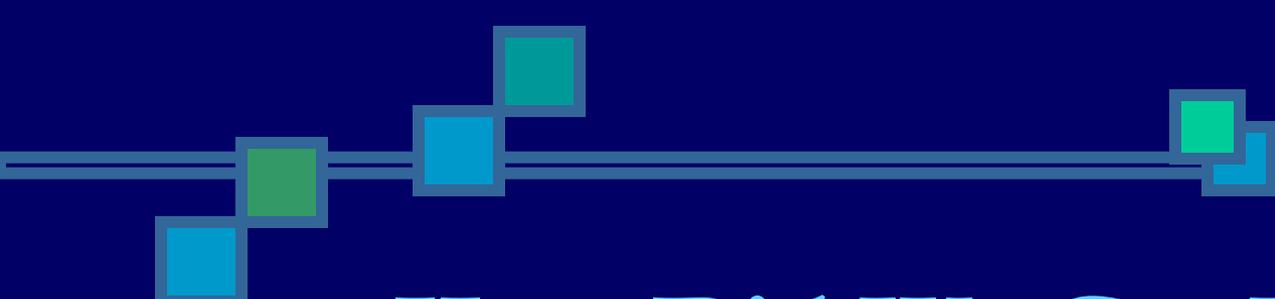


- ✓ **Aerospace Application Robustness**

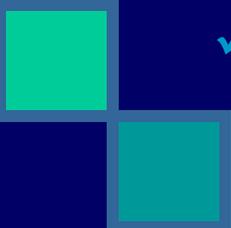
- System with Uncertain Description & Structure

- ✓ **Model-Reality Mismatch**

- Identification of a Known Linear Model
  - Robust Residual Generation
- 



# How Did We Get Here? (i)

- 
- ✓ The analytical process model is derived through an identification algorithm as the accurate analytical model is not achievable from physical principles.
  - ✓ The MEX system is not perfectly known & the model parameters can be provided only with a certain degree of accuracy.
- 



# How Did We Get Here? (ii)

✓ In practice:

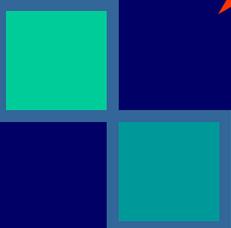
- the determination of a mathematical model of the dynamic system under investigation.
- the design of the diagnosis scheme.

are considered by means of direct analysis of the input-output time series data.

✓ The *proposed method* solves at the same time the system identification & the residual generator design problems.



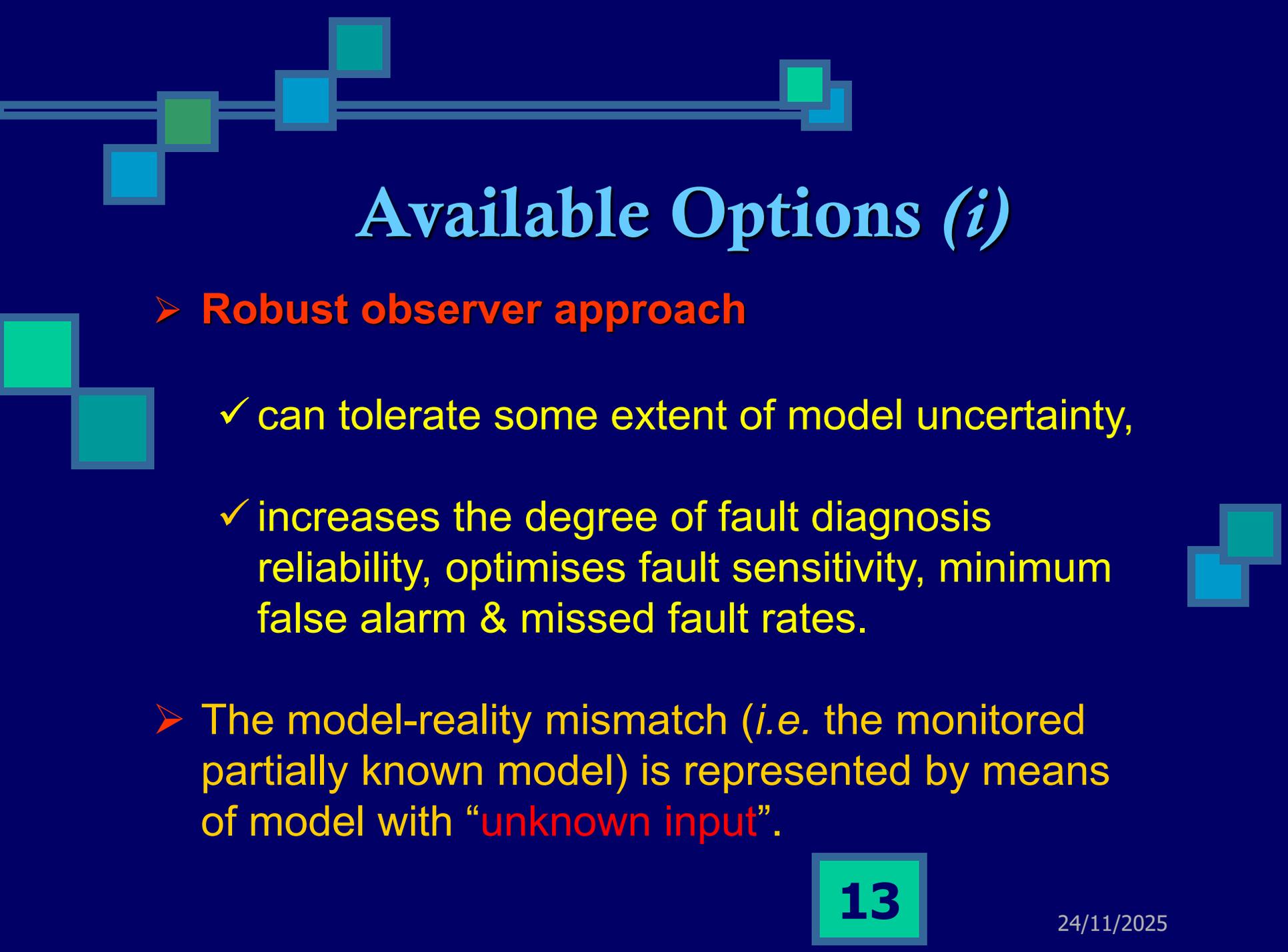
## How Did We Get Here? *(iii)*

- 
- The motivation for this combined procedure is based on requirements for:
    - ✓ Robust discrimination between uncertainty (model-reality mismatch) and faults;
    - ✓ Sensitivity to faults for good fault isolation.
- 



# How Did We Get Here?

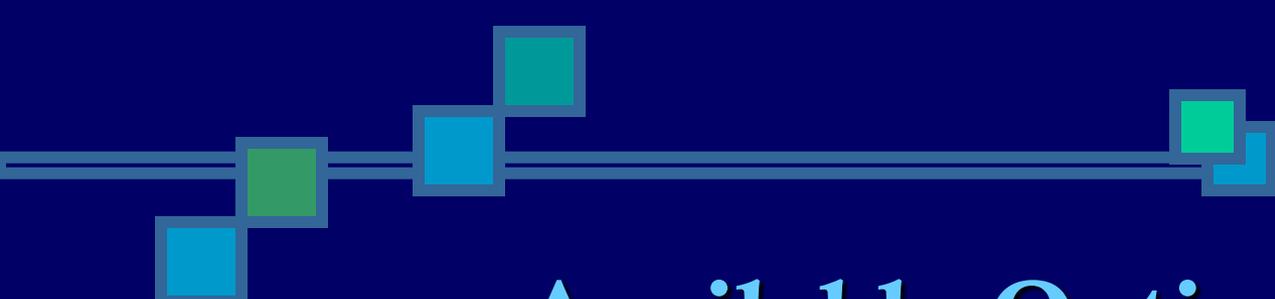
- ✓ A perfectly accurate mathematical model of a physical system is not available.
  - ✓ The parameters of the system may vary with time & the characteristics of the disturbance & noises are unknown.
  - ✓ Mismatch between the actual process and its mathematical model (*model-mismatch error*).
  - ✓ Disturbance signals act as sources of *false alarms and missed alarms*.
- 



# Available Options (*i*)

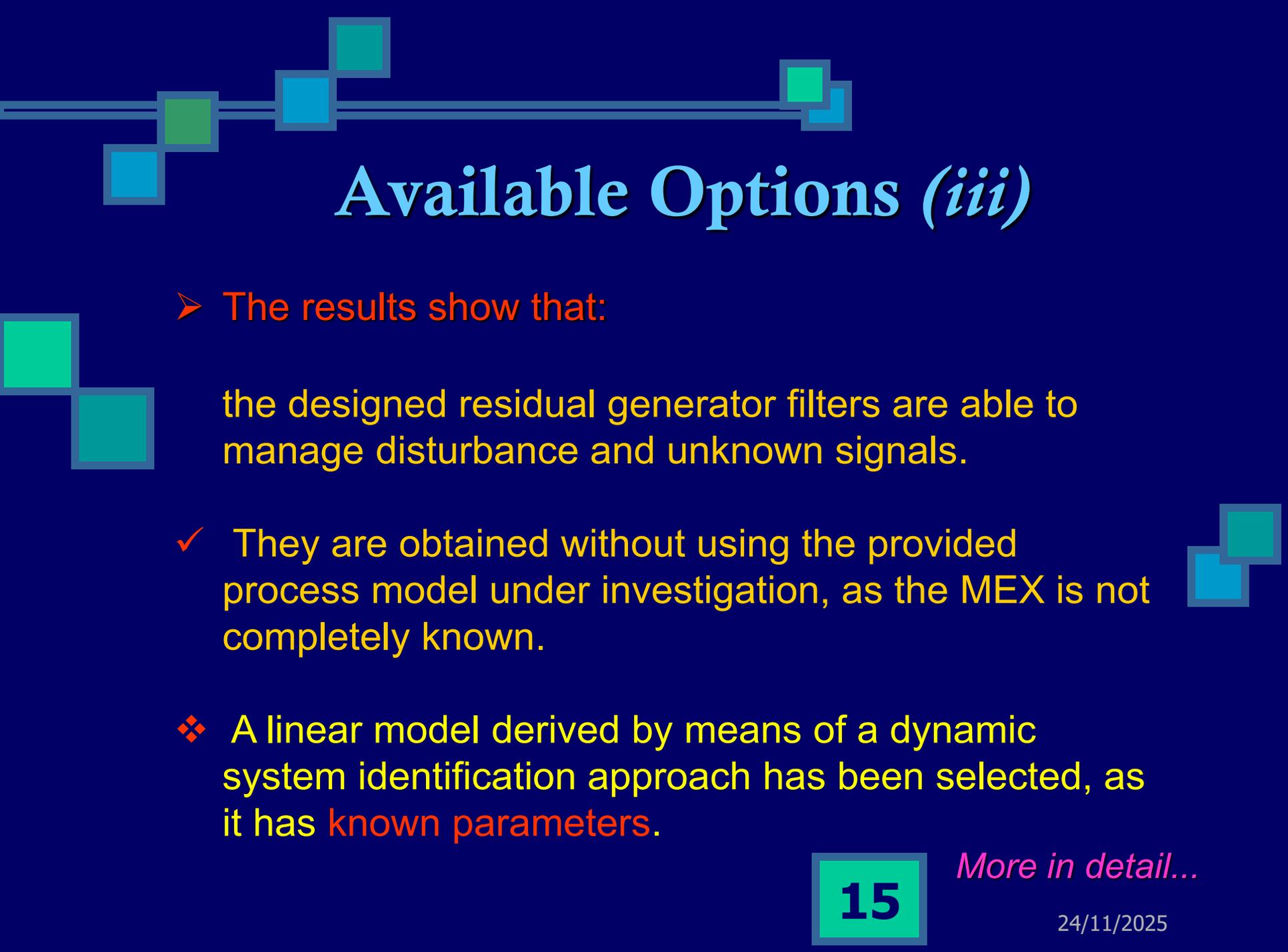
## ➤ Robust observer approach

- ✓ can tolerate some extent of model uncertainty,
  - ✓ increases the degree of fault diagnosis reliability, optimises fault sensitivity, minimum false alarm & missed fault rates.
- The model-reality mismatch (*i.e.* the monitored partially known model) is represented by means of model with “unknown input”.



## Available Options *(ii)*

- ✓ Identification of a linear model of the monitored system (with unknown or partially known parameters).
  - ✓ Design of residual generators for a linear multivariable system with additive faults and disturbance signals.
  - ✓ The process under investigation is modelled in terms of input–output or state-space descriptions.
  - ✓ Straightforward design of the residual generation functions implemented by means of dynamic observers or filters.
- 



# Available Options (iii)

➤ The results show that:

the designed residual generator filters are able to manage disturbance and unknown signals.

- ✓ They are obtained without using the provided process model under investigation, as the MEX is not completely known.
- ❖ A linear model derived by means of a dynamic system identification approach has been selected, as it has known parameters.

## Available Options (4)

$$\begin{cases} x_p(t+1) = A_p(\theta) x_p(t) + B_p(\theta) u(t) \\ y(t) = C_p(\theta) x(t) \end{cases}$$

**MEX  
Process**


$$\begin{cases} x(t+1) = A x(t) + B u(t) + H e(t) \\ y(t) = C x(t) + e(t) \end{cases}$$

**Identified  
System**

## Available Options (5)

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) + H(y(t) - Cx(t)) \\ y(t) = Cx(t) \\ e(t) = y(t) - Cx(t) \\ r(t) = e(t) \end{cases}$$

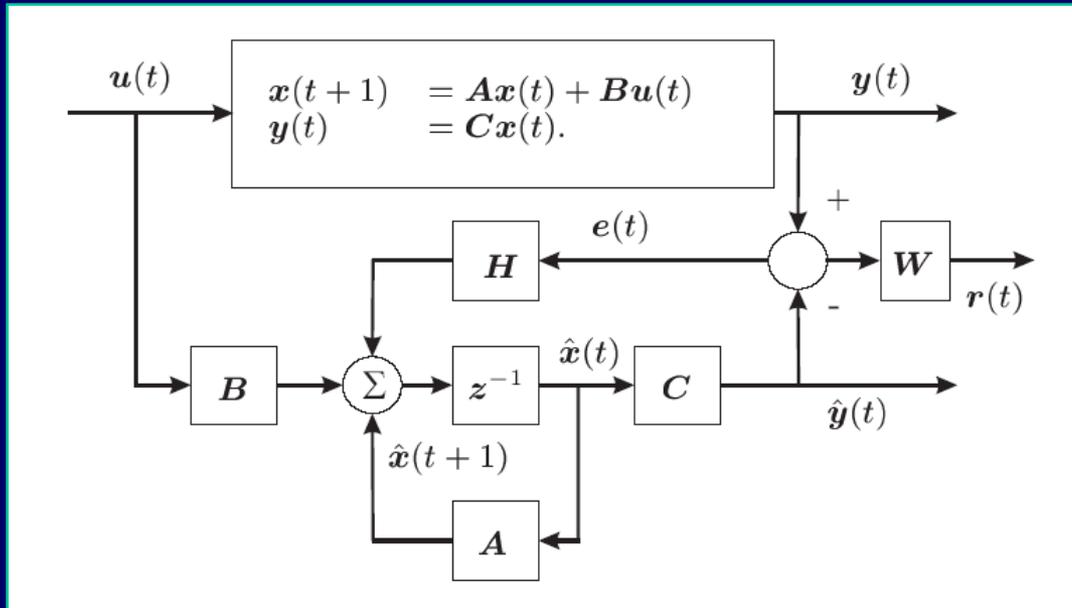
**Residual  
Generation**

➤ **Performance criteria optimisation:**

✓ **Matrices identification (A, B, C, H)**

✓ **Minimisation of  $\|e(t)\|_2^2$**

# Available Options (6)



**Residual  
Generation**

$$\min_H \|e(t)\|_2^2$$

$\begin{cases} |r(t)| \leq \varepsilon, \text{ in the fault-free case} \\ |r(t)| > \varepsilon, \text{ in the faulty case} \end{cases}$

**Residual  
Evaluation**

# Available Options (7)

(i)

$$(1 - th) \times \min(r_h(t)) \leq r_f(t) \leq (1 + th) \times \max(r_h(t))$$

for the fault - free case

$$r_f(t) < (1 - th) \times \min(r_h(t)) \text{ or } r_f(t) > (1 + th) \times \max(r_h(t))$$

for the faulty case

**Residual  
Evaluation**

(ii)

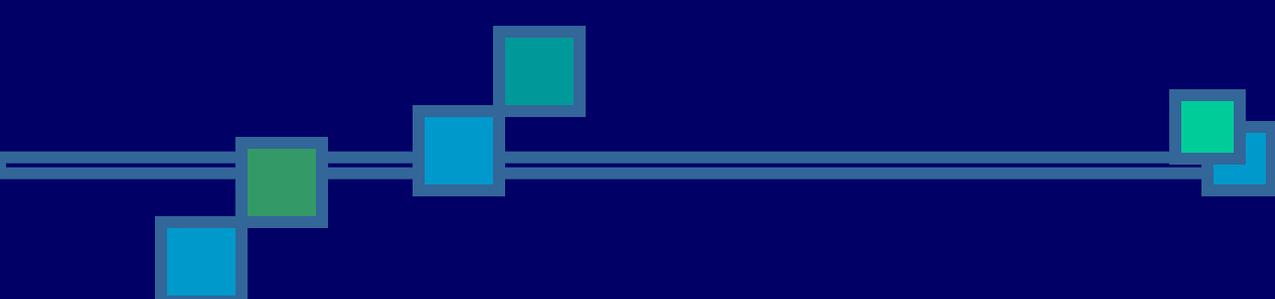
$$mean(r_h(t)) - \sigma \times std(r_h(t)) \leq r_h(t) \leq mean(r_h(t)) + \sigma \times std(r_h(t))$$

for the fault - free case

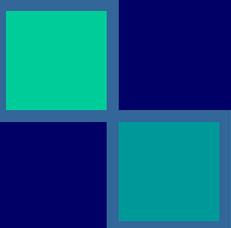
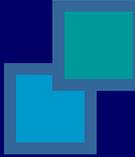
$$r_f(t) > mean(r_h(t)) + \sigma \times std(r_h(t)) \text{ or } r_f(t) < mean(r_h(t)) - \sigma \times std(r_h(t))$$

for the faulty case

(Adaptive thresholds? ...)



# Recommendation

- 
- ✓ *Proposed Method* General Introduction
  - ✓ Identification, Design & Tuning Methodology
  - ✓ Result Analysis
  - ✓ Good performances in simulation
- 

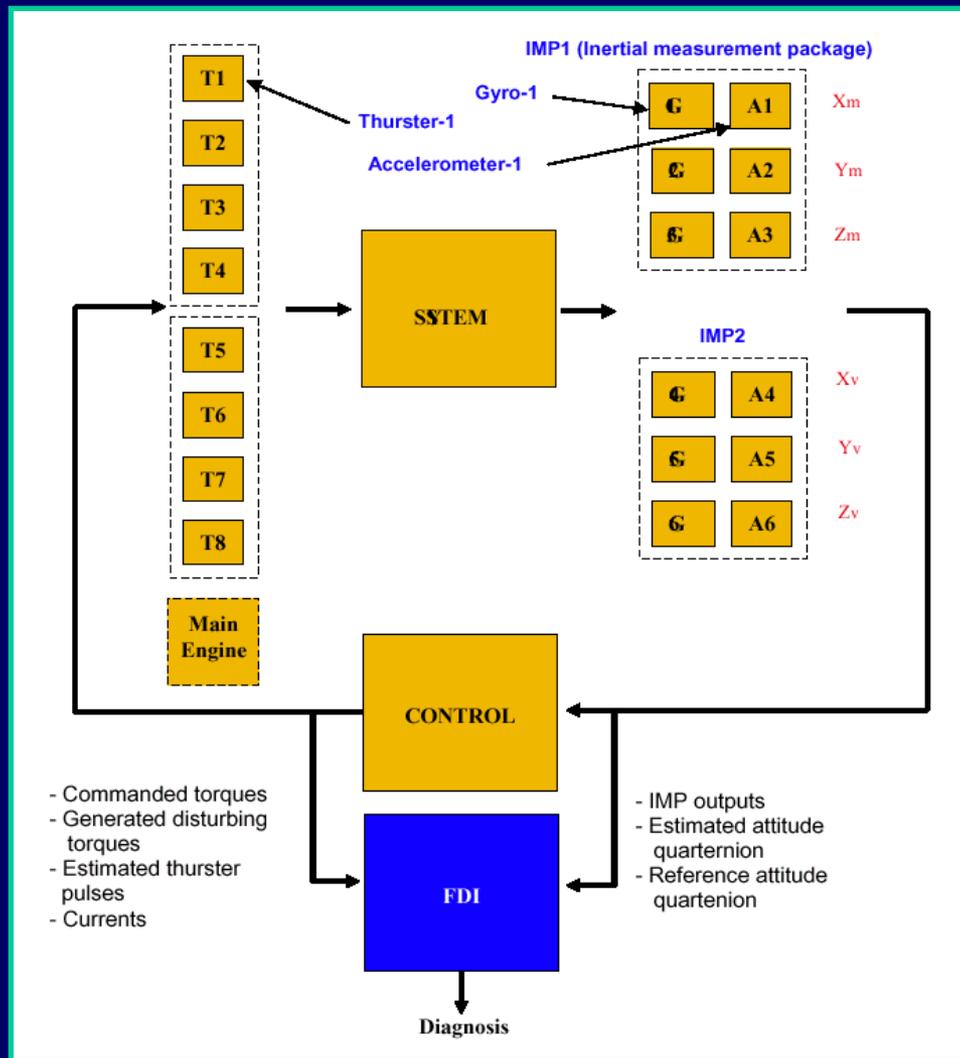
➤ Detailed Procedure Description 



## Identification, Design & Tuning Methodology

- ✓ Process Description
  - ✓ Dynamic System Identification
  - ✓ EE and EIV Model Identification
  - ✓ System Identification for Robust FDI
  - ✓ FDI Logic Scheme
  - ✓ The Robustness Problem of Residual Generation for FDI
  - ✓ Observer-based Approaches for FDI
  - ✓ The Residual Generation Problem
- 

# Monitored MEX Process



**Identification**  
+  
**Residual Generation**  
+  
**Residual Evaluation**  
+  
**Performance Evaluation**

# Process Description

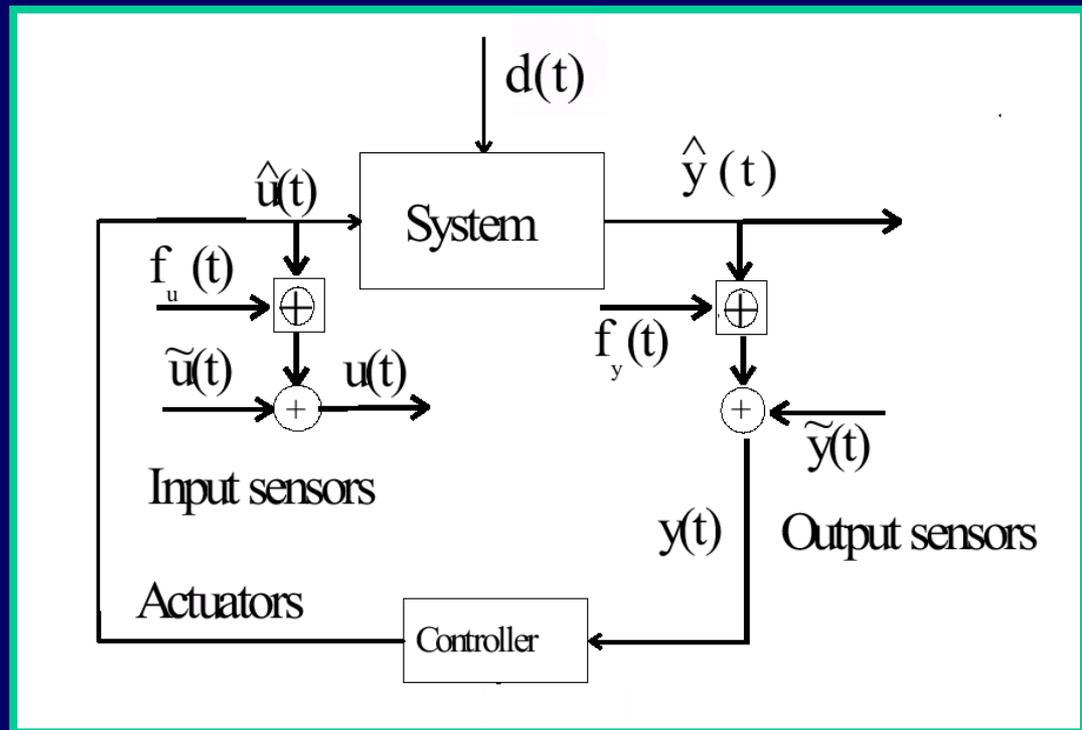
$$\begin{cases} \mathbf{x}_p(t+1) = A_p \mathbf{x}_p(t) + B_p \hat{\mathbf{u}}(t) \\ \hat{\mathbf{y}}(t) = C_p \mathbf{x}_p(t), \quad t = 1, 2, \dots, N \end{cases}$$

State-Space Model



$$\begin{cases} \mathbf{u}(t) = \hat{\mathbf{u}}(t) + \tilde{\mathbf{u}}(t) + \mathbf{f}_u(t) \\ \mathbf{y}(t) = \hat{\mathbf{y}}(t) + \tilde{\mathbf{y}}(t) + \mathbf{f}_y(t) \end{cases}$$

Fault & disturbance signals



# Dynamic System Identification (i)

$$\begin{cases} \mathbf{u}(t) = \hat{\mathbf{u}}(t) + \tilde{\mathbf{u}}(t) \\ \mathbf{y}(t) = \hat{\mathbf{y}}(t) + \tilde{\mathbf{y}}(t) \end{cases}$$

$$\hat{\mathbf{y}}(t) = \sum_{i=1}^m \sum_{k=1}^n \alpha_{ik} \hat{\mathbf{y}}_i(t-k) + \sum_{j=1}^r \sum_{k=1}^n \beta_{ikj} \hat{\mathbf{u}}_j(t-k) + e(t)$$

Input-output ARX/EIV models

$$\begin{cases} \mathbf{x}(t+1) = \mathbf{A} \mathbf{x}(t) + \mathbf{B} \mathbf{u}(t) + \mathbf{H} e(t) \\ \mathbf{y}(t) = \mathbf{C} \mathbf{x}(t) + e(t), \quad t = 1, 2, \dots \end{cases}$$

*Innovation form* state-space model

# Dynamic System Identification (2a) PEM

## From input-output to (canonical) state-space models

$$\mathbf{A} = [\mathbf{A}_{ij}],$$

$$\mathbf{A}_{ii} = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ \alpha_{ii1} & \alpha_{ii2} & \dots & \alpha_{ii\nu_i} \end{bmatrix}_{(\nu_i \times \nu_i)},$$

$$\mathbf{A}_{ij} = \begin{bmatrix} 0 & \dots & \dots & \dots & \dots & 0 \\ \vdots & & & & & \vdots \\ 0 & \dots & \dots & \dots & \dots & 0 \\ \alpha_{ij1} & \dots & \alpha_{ij\nu_j} & 0 & \dots & 0 \end{bmatrix}_{(\nu_i \times \nu_j)}$$

$$\mathbf{B} = \mathbf{M}^{-1}\bar{\mathbf{B}}$$

where

$$\bar{\mathbf{B}} = \begin{bmatrix} \bar{\mathbf{B}}_1 \\ \bar{\mathbf{B}}_2 \\ \vdots \\ \bar{\mathbf{B}}_m \end{bmatrix}, \quad \bar{\mathbf{B}}_i = \begin{bmatrix} \beta_{i11} & \dots & \beta_{i\nu_i 1} \\ \vdots & \ddots & \vdots \\ \beta_{i1\nu_i} & \dots & \beta_{i\nu_i \nu_i} \end{bmatrix},$$

and

$$\mathbf{M} = [\mathbf{M}_{ij}], \text{ with } i, j = 1, \dots, m$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & 1 & 0 & \dots & \dots & 0 \\ \vdots & \dots & \dots & \dots & \dots & \dots & \dots & \vdots \\ 0 & \dots & \dots & \dots & 0 & 1 & 0 & \dots & 0 \end{bmatrix}$$

where

$$\mathbf{M}_{ii} = \begin{bmatrix} -\alpha_{ii2} & -\alpha_{ii3} & \dots & -\alpha_{ii\nu_i} & 1 \\ -\alpha_{ii3} & -\alpha_{ii4} & \dots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ -\alpha_{ii\nu_i} & 1 & & \vdots & \vdots \\ 1 & & & & \vdots \end{bmatrix}_{(\nu_i \times \nu_i)}$$

and

$$\mathbf{M}_{ij} = \begin{bmatrix} -\alpha_{ii2} & -\alpha_{ii3} & \dots & -\alpha_{ii\nu_i} & 0 \\ -\alpha_{ii3} & -\alpha_{ii4} & \dots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ -\alpha_{ii\nu_i} & 0 & & \vdots & 0 \\ 0 & \dots & \dots & \dots & 0 \\ 0 & \dots & \dots & \dots & 0 \end{bmatrix}_{(\nu_i \times \nu_j)}$$

# Dynamic System Identification (2b) *N4SID*

Numerical algorithm for Subspace State-Space System Identification

$$O_i \equiv \frac{Y_f}{U_f} \begin{bmatrix} U_p \\ Y_p \end{bmatrix}$$

“Oblique projection”

$$O_i = [U_1 \quad U_2] \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

SVD

$$\Gamma_i = U_1 S^{1/2}$$

$$A = \Gamma_i^+ \Gamma_i \quad \dots$$

Matrices  $(A, B, C, H)$  computation

# Dynamic System Identification

(3)

$$\begin{cases} \mathbf{x}_p(t+1) = A_p \mathbf{x}_p(t) + B_p \hat{u}(t) \\ \hat{y}(t) = C_p \mathbf{x}_p(t), \quad t = 1, 2, \dots, N \end{cases}$$

From uncertain system to known model with unknown inputs

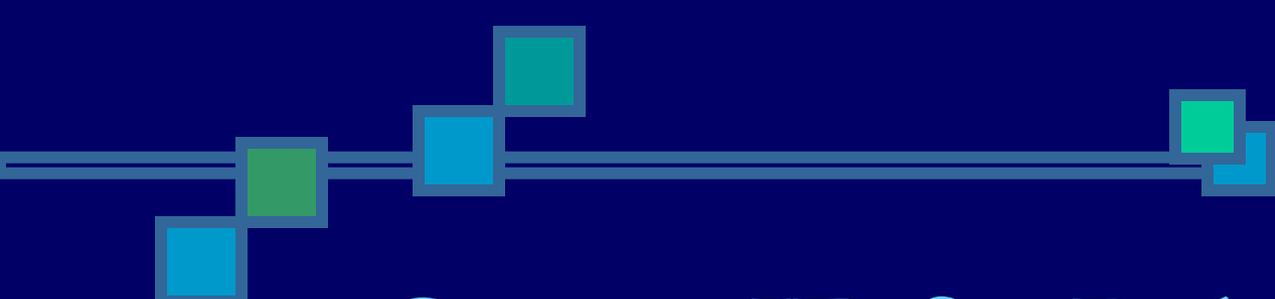
$$\begin{cases} \mathbf{x}(t+1) = A \mathbf{x}(t) + B \mathbf{u}(t) + H e(t) \\ \mathbf{y}(t) = C \mathbf{x}(t) + e(t), \quad t = 1, 2, \dots \end{cases}$$

Innovation form state-space model

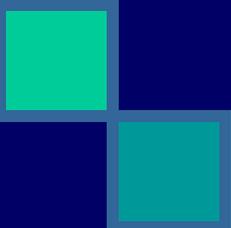
$$\text{s.t. } e(t) = y_p(t) - y(t) \approx 0$$

$$\min_H \|e(t)\|_2^2$$

$$(A, B, C, H)$$



# System ID for Robust FDI

- 
- ✓ Technique insensitive to modelling uncertainty.
  - ✓ Identification of the “disturbance distribution matrix” i.e. the matrix  $H$ .
  - ✓ The uncertain MEX system is described by means of a known model with unknown inputs  $e(t)$ .
  - ✓ The residual generator can generate residual signals with disturbance de-coupling properties:
- Robust FDI solution.
- 

# Observer-based Approach for FDI

Identified  
model

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) + He(t) \\ y(t) = Cx(t) + e(t) \end{cases}$$

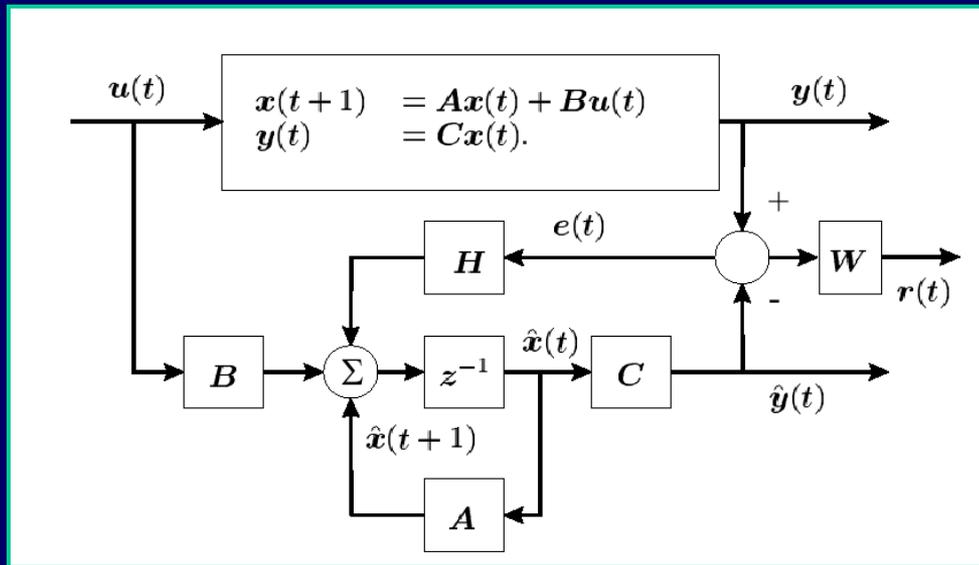
Diagnostic  
observer

$$\begin{cases} \hat{x}(t+1) = A\hat{x}(t) + Bu(t) + He(t) \\ e(t) = y(t) - C\hat{x}(t) \end{cases}$$

**Residual Generation**

*(Innovation form)*

# Residual Generation (i)



## Properties

$$\begin{cases} e_x(t) = x(t) - \hat{x}(t) \\ e_x(t+1) = (A - HC) e_x(t) \end{cases}$$

$$\lim_{t \rightarrow \infty} e_x(t) = 0$$

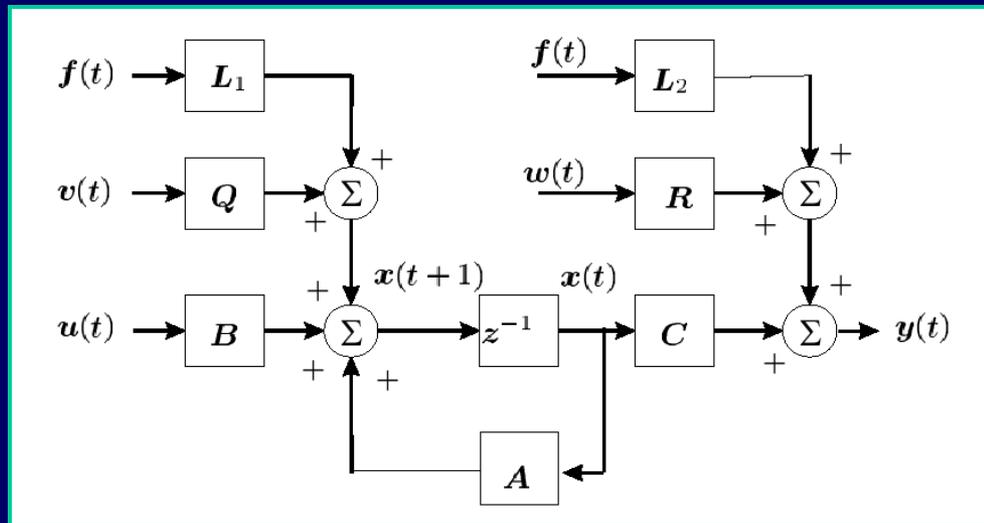
$$r(t) = W e(t) = W (y(t) - C \hat{x}(t)) = W (y(t) - \hat{y}(t))$$

## Residual Properties

# Residual Generation (ii)

$$\begin{cases} x(t+1) = A x(t) + B u(t) + Q v(t) + B f(t) \\ y(t) = C x(t) + R w(t) + f(t) \end{cases}$$

$$e_x(t+1) = (A - H C) e_x(t) + L_1 f(t) - H L_2 f(t)$$



$$e(t) = C e_x(t) + L_2 f(t)$$

**Residual generation problem**

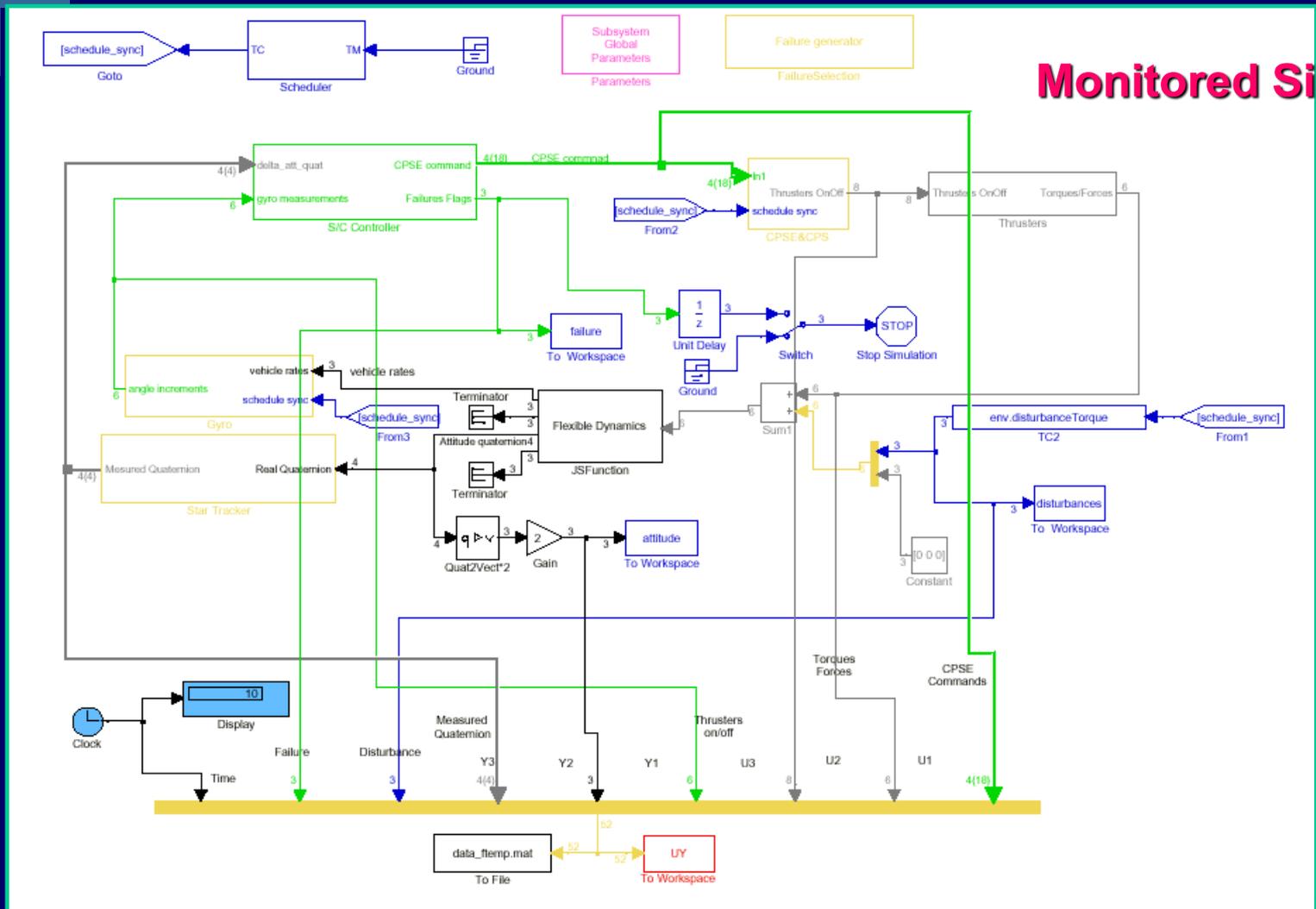
# Residual Generation Problem

- ✓ Uncertainty = Disturbance  $\Rightarrow$  Unknown Input  $e(t)$
- ✓ Disturbance de-coupling  $\Rightarrow \min_H |e(t)|$
- ❖ Fault sensitivity and disturbance robustness trade-off = *optimisation problem (literature)*
- ❖ Disturbance effect minimisation (*this method*)
- Identification + robust filter design ( $H_2$ )

# Analytical Performance Analysis

- ✓ Application Example and Results
  - The MEX Satellite System Simulator →
- ✓ Simulated Fault Conditions
- ✓ MEX State-Space Model and MEX Model Structure for FDI
- ✓ Some Practical Advices for State-Space Model Building
- ✓ Residual Generator Design for FDI – Dynamic Observers
- ✓ Fault Isolability
- ✓ Comments, suggestions & further works

# Analytical Performance Analysis (2)



**Monitored Signals**



# Analytical Performance Analysis

(3)

- ✓ CPSE Commands (18 signals): Command flag, Delay, Duration & Mode;
- ✓ Torque forces (6 signals);
  - Thruster on/off (4/8 signals);
  - Gyro measurements (6 signals);
- ✓ Attitude (3 signals);
  - Measured quaternions (4 signals);
- ✓ Disturbance (3 signals);
- ✓ Failure (3 signals).

System Identification



❖  $N=10000$  samples &  $T_s=0.1$  seconds.

# Analytical Performance Analysis

(4)

➤ Identified model performances

$$VAF\% = 100 \times \frac{1 - \text{std}(y(t) - \hat{y}(t))}{\text{std}(y(t))}$$

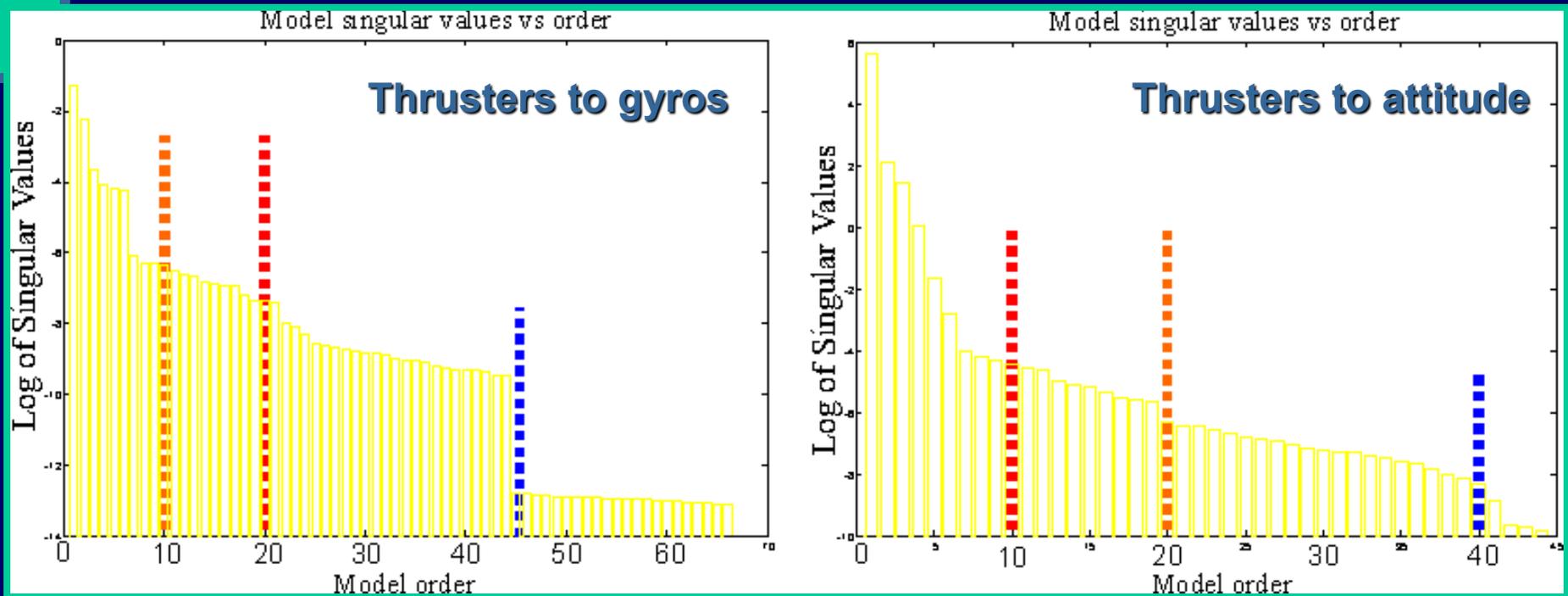
Model Order Selection →

Type	Inputs	Outputs	Order	VAF (Ident.)	VAF (Valid.)
SS	CPSE Commands	Gyro Measurements	20	89,96%	69,50%
SS	Thrusters On/Off	Gyro Measurements	18	82,05%	72,45%
SS	Thrusters On/Off	Gyro Measurements	20	92,99%	72,09%
ARX	Thrusters On/Off	Gyro Measurements	20	93,65%	73,15%
ARX	CPSE Commands	Gyro Measurements	18	84,63%	68,93%
SS	CPSE Commands	Gyro Measurements	20	96,15%	66,95%
ARX	Thrusters On/Off	Measured Quaternions	20	80,79%	67,59%
ARX	Thrusters On/Off	Measured Quaternions	20	78,55%	69,95%
ARX	Thrusters On/Off	Measured Quaternions	18	70,58%	60,98%

# Analytical Performance Analysis

(5)

➤ Identified model order selection



✓ PPCRE% or VAF% criteria

(n4sid tool)

# Analytical Performance Analysis

## (6) Simulated fault conditions

$N^{\circ}$	Fault Case	Symbol
1.	No-fault	$h$
2.	Thruster 2 Open	$f_1$
3.	Thruster 2 Closed	$f_2$
4.	Thruster 4 Open	$f_3$
5.	Thruster 4 Closed	$f_4$
6.	Gyro 1 Null	$f_5$
7.	Gyro 1 Drift	$f_6$
8.	Gyro 1 Const.	$f_7$

- Fault condition detection with minimum delay;
- Fault location identification (isolation) detection with minimum delay;
- ❖ In the presence of disturbance.

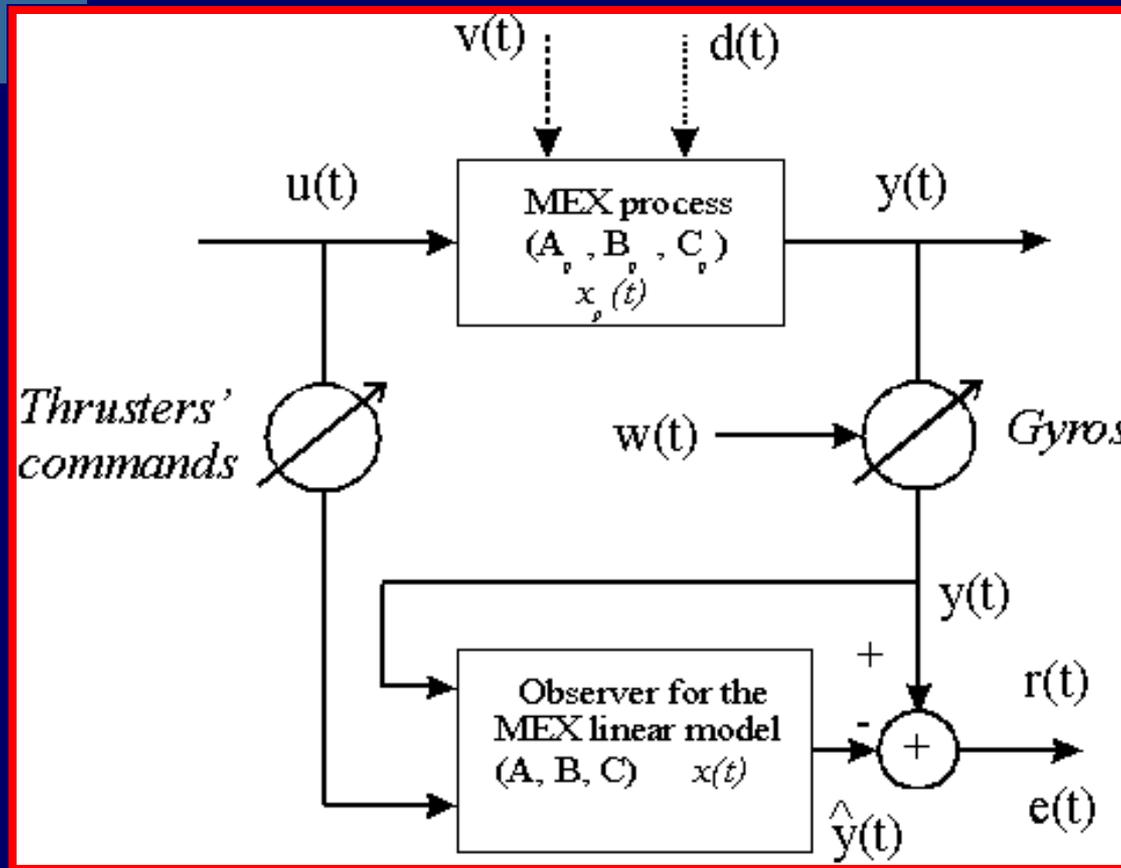
# Analytical Performance Analysis (7a)

- ✓ The MEX *non-linear* plant is a 18<sup>th</sup> order model when flexible modes are considered (study first stage);
- ✓ The simplified MEX *linear* model is a 6<sup>th</sup>/7<sup>th</sup> order system when the flexible modes (due to solar cell arrays) can be neglected (in the short transient);
- ✓ The MEX system can be considered linear when the behavior from thrusters' forces to attitude measurements are considered; small pointing errors (study second stage);
- ✓ The model of the thrusters is non-linear, *i.e.* when the actuators' link between thrusters' forces and thrusters' on/off commands have to be taken into account;

# Analytical Performance Analysis (7b)

- ✓ The MEX model can be simulated in different working conditions;
- ✓ Fault detectability and MEX model structure can change with time;
- ✓ Disturbance and uncertain signals are present; they represent main engine and thruster misalignment;
- ✓ Disturbance is a torque orthogonal to the direction (a force);
- ✓ The dynamics of the rigid body (*i.e.* when the MEX satellite is considered for ) is well-known; the model of the flexible models (acting for and due to solar cell array flexible structure) is not known;
- ✓ An estimation of the disturbance torques is available from the Simulink MEX model; this estimation cannot be reliable in long time due also to noise signals.

# Analytical Performance Analysis (8)



$$J = \frac{1}{N} \det \left( \sum_{t=1}^N e(t) e^T(t) \right)$$



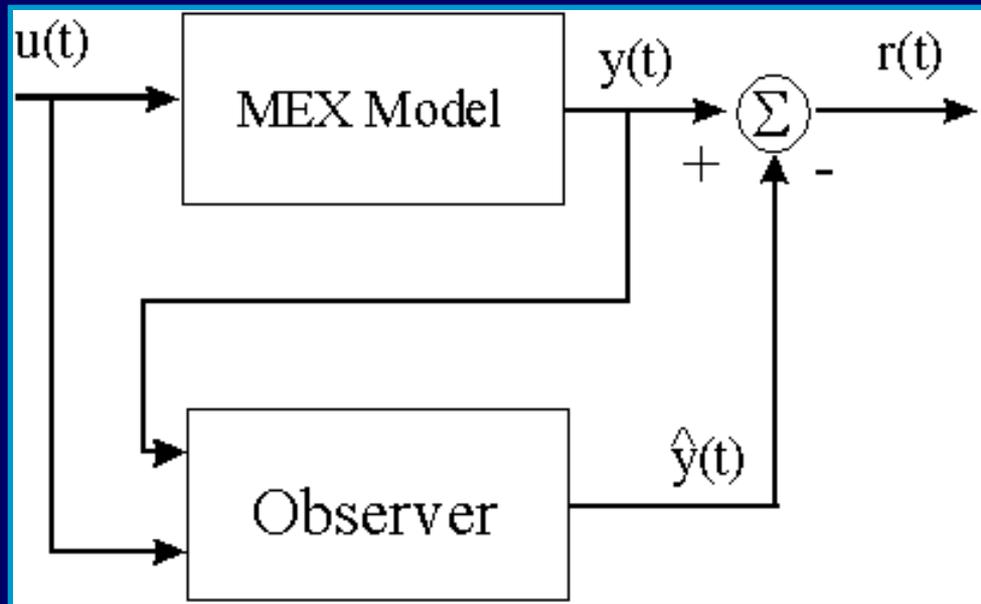
$(A, B, C \text{ and } H)$



$$\begin{cases} x_p(t+1) = A_p x_p(t) + B_p u(t) + E_p d(t) + v(t) \\ y(t) = C x_p(t) + w(t) \end{cases}$$

$$\begin{cases} x(t+1) = A x(t) + B u(t) + H e(t) \\ y(t) = C x(t) + e(t) \end{cases} \quad x_0 = x(0)$$

# Analytical Performance Analysis (9)



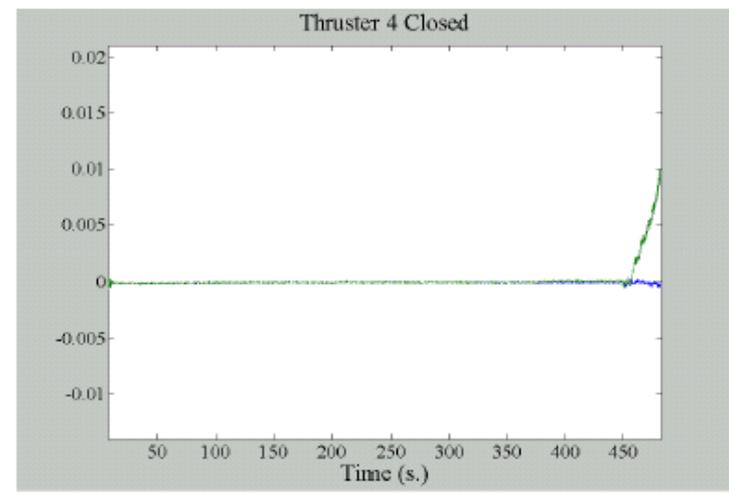
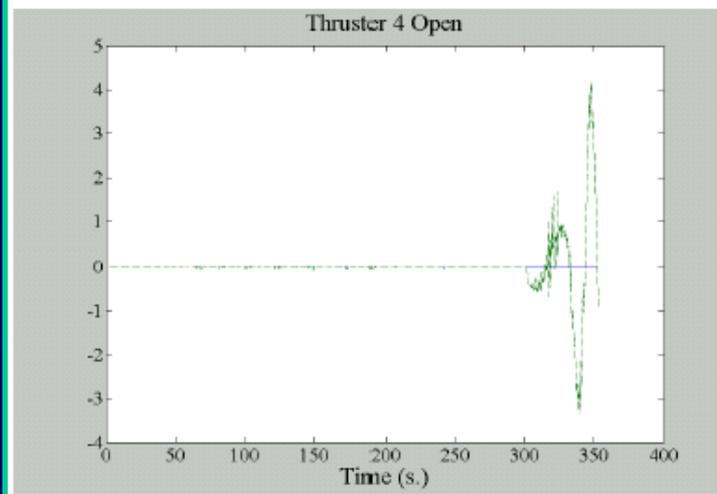
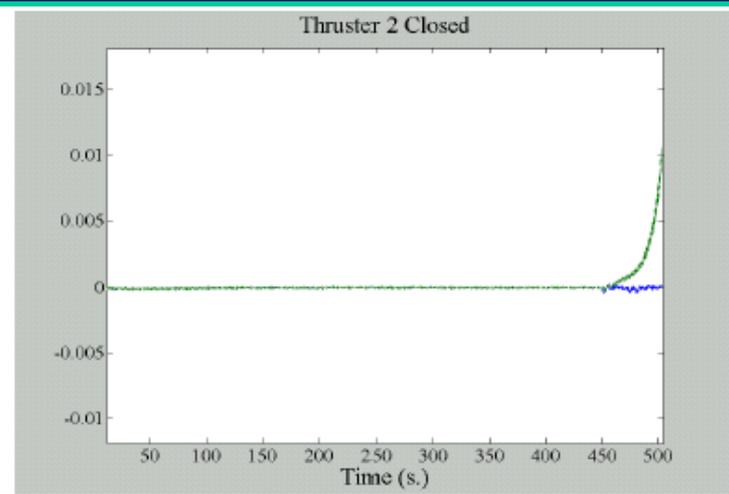
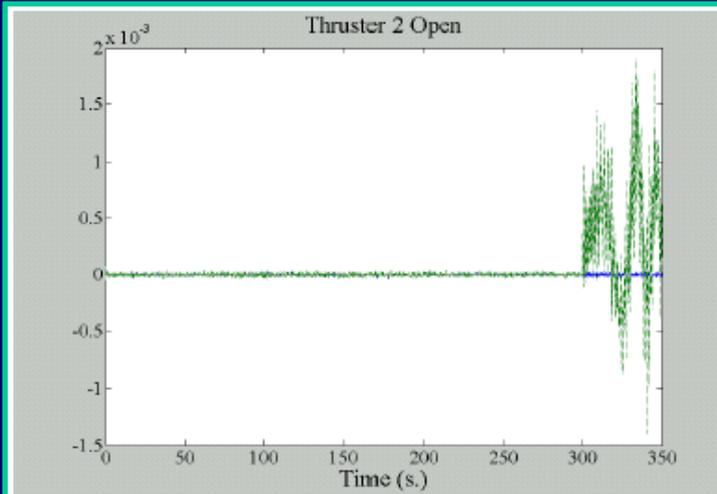
Observer  
design

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) + H(\hat{y}(t) - y(t)) \\ y(t) = Cx(t) \\ r(t) = Qe(t) = Q(y(t) - \hat{y}(t)) \end{cases}$$

$$r(t) = Qe(t) = Q(y(t) - \hat{y}(t))$$

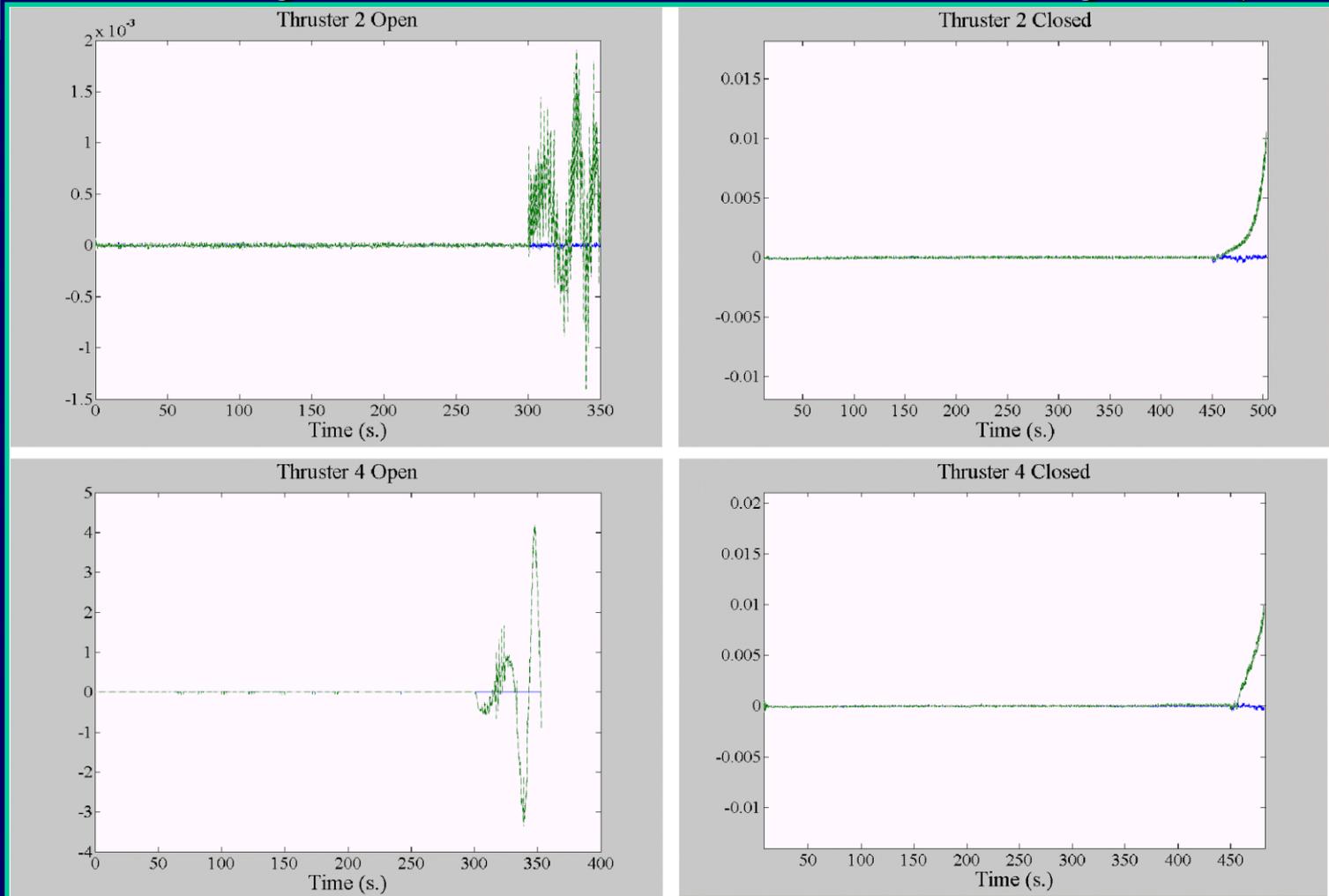
Residual Generation for  
Fault Detection

# Analytical Performance Analysis (10)



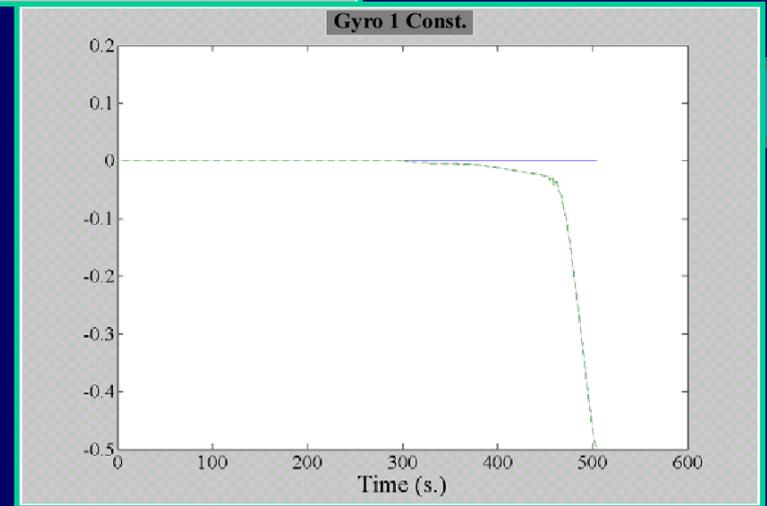
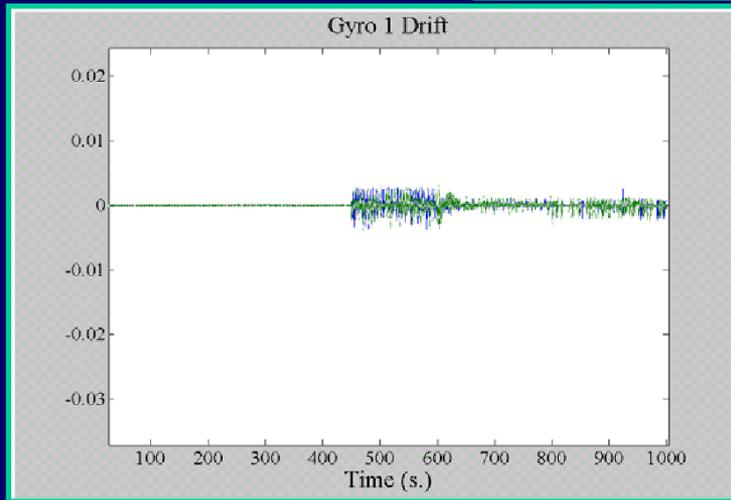
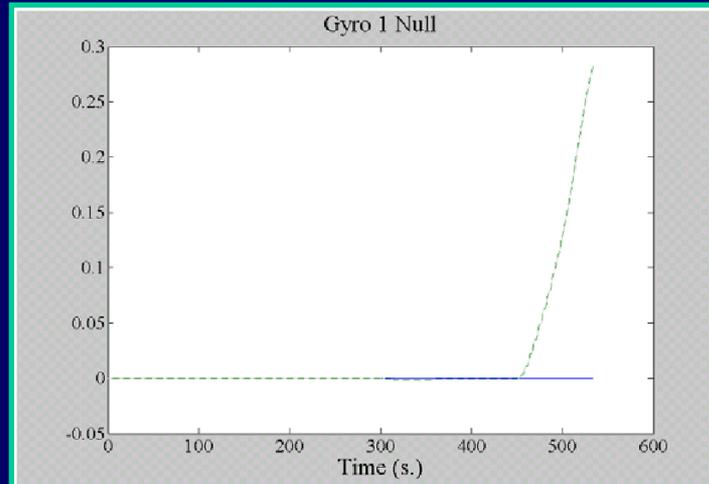
Fault-free & faulty residuals for thruster faults.

# Analytical Performance Analysis (10a)



Fault-free & faulty residuals for thruster faults.

# Analytical Performance Analysis (11)



The fault-free & faulty residuals for gyro 1 faults.

# Analytical Performance Analysis (12)

## Fault Isolation

- "Structured" Residual Selection
  - Depend on  $(A, B, C, H)$  model structure

$$r_i(t) = \max_i \|r_i(t)\|_2^2 \quad (\text{with } i = 1, \dots, m)$$

*Voting scheme*

- ✓  $r_i(t)$  represents the faulty residual vector
- "Fault Signature Matrix" Generation →
- ❖ Given detectability/isolability properties

# Analytical Performance Analysis (13)

## ✓ Fault Isolation Table: Thruster faults (1)

Table 4: Fault signature with 3 residual signals.

Fault Case	$r_1(t)$	$r_2(t)$	$r_3(t)$	Detection Delay
Thruster 2 Open	1	1	0	~ 0.1 s.
Thruster 2 Closed	0	1	0	~ 20 s.
Thruster 4 Open	0	1	1	~ 0.6 s.
Thruster 4 Closed	1	0	1	~ 50 s.

## ✓ Distinguishable table rows

❖ 3 residual signals suitably selected!

# Analytical Performance Analysis (14)

## ✓ Fault Isolation Table: Thruster faults (2)

Table 3: Fault signature with 4 residual signals.

Fault Case	$r_1(t)$	$r_2(t)$	$r_3(t)$	$r_4(t)$	Detection Delay
Thruster 2 Open	0	1	0	1	~ 0.4 s.
Thruster 2 Closed	1	0	1	0	~ 10 s.
Thruster 4 Open	0	1	0	1	~ 0.8 s.
Thruster 4 Closed	1	0	0	1	~ 30 s.

## ✓ 2 Undistinguishable table rows

❖ 4 residual signals were selected

# Analytical Performance Analysis (15)

## ✓ Fault Table: Gyro faults

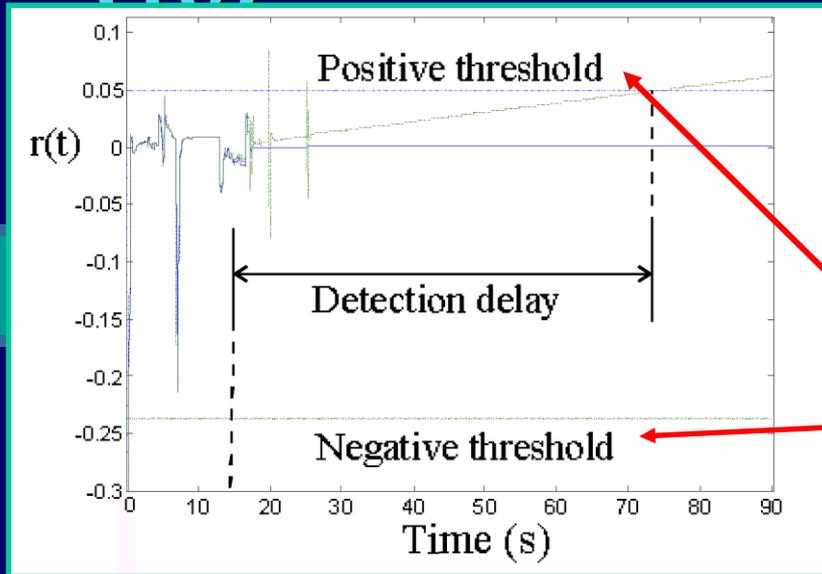
Table 5: Detection delays for the gyro fault cases.

Fault Case	Detection Delay
Gyro 1 Null	~ 50 s.
Gyro 1 Drift	~ 40 s.
Gyro 1 Const.	~ 70 s.

➤ Detection Delay Time ➡

# Analytical Performance Analysis

(16)



$$\begin{cases} (1 - th) \times \min(r_h(t)) \leq r_f(t) \leq (1 + th) \times \max(r_h(t)) \\ \text{for the fault - free case} \\ r_f(t) < (1 - th) \times \min(r_h(t)) \text{ or } r_f(t) > (1 + th) \times \max(r_h(t)) \\ \text{for the faulty case} \end{cases}$$

Parameters  $th$  &  $\sigma$

✓ Detection or isolation delay-time evaluation

➤ Adaptive thresholds: (?)

$$\begin{cases} \text{mean}(r_h(t)) - \sigma \times \text{std}(r_h(t)) \leq r_h(t) \leq \text{mean}(r_h(t)) + \sigma \times \text{std}(r_h(t)) \\ \text{for the fault - free case} \\ r_h(t) > \text{mean}(r_h(t)) + \sigma \times \text{std}(r_h(t)) \text{ or } r_h(t) < \text{mean}(r_h(t)) - \sigma \times \text{std}(r_h(t)) \\ \text{for the faulty case} \end{cases}$$

$$r(t) \equiv \Psi(t, r_h(t), u(t))$$

# Analytical Performance Analysis (17)

## ❖ Simulation Analysis

- *False Alarm Rate;*
- *Correct Isolation Rate;*
- *Wrong Isolation Rate;*
- *Missed Fault Rate;*
- *Mean Detection Time;*
- *Mean Isolation Time;*
- *Fault Alarm Rate;*
- *Fault Detection Probability;*
- *Fault Isolation Probability;*
- *Computation Time;*
- *CPU Time.*

Evaluated parameters  
(over simulation experiments)

### Parameters $th$ & $\sigma$

- The false alarm rate;
- The missed alarm rate;
- The wrong isolation rate;
- Detection delay time;
- Isolation delay time;
- CPU computation time;

Minimisation

- The correct fault alarm rate;
- The correct fault isolation rate;

Maximisation

# Analytical Performance Analysis (18)

## ❖ Tuning Methodology.

### ✓ Initial settling of $th$ and $\sigma$ parameters.

➤ Empirically fixed, *e.g.*:

- $th = 0.1$  (+/- 10% of max & min fault-free  $r(t)$ ).
- $\sigma = 3$  ( $3\sigma$  law for 99% confidence level WGN).

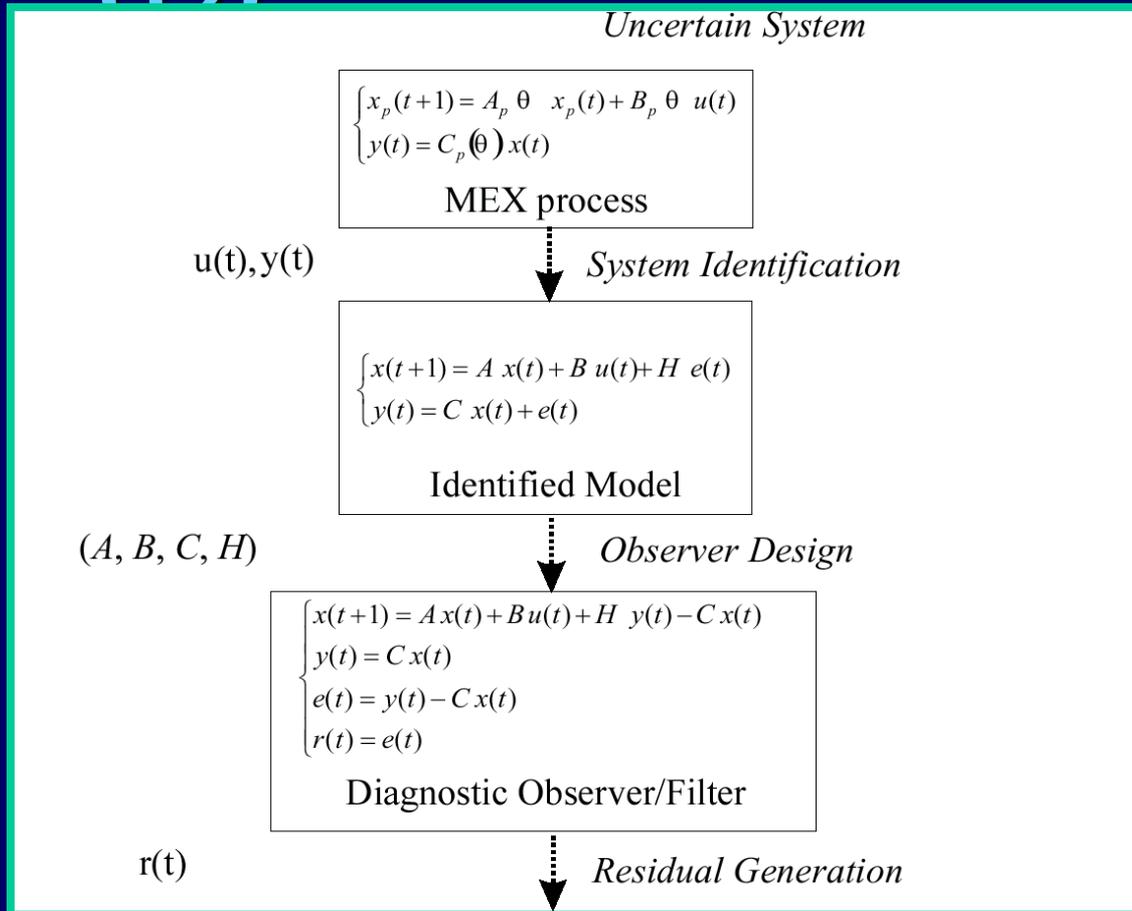
### ✓ Fine parameter tuning.

✓ Simulation analysis with experiments.

▪ Parameter validation (using a different data set).

❖ *Fault sensitivity maximisation, false alarm & missed fault alarm rate minimisation* (Monte-Carlo analysis  $N_{MC}$ ).

# Analytical Performance Analysis (19)

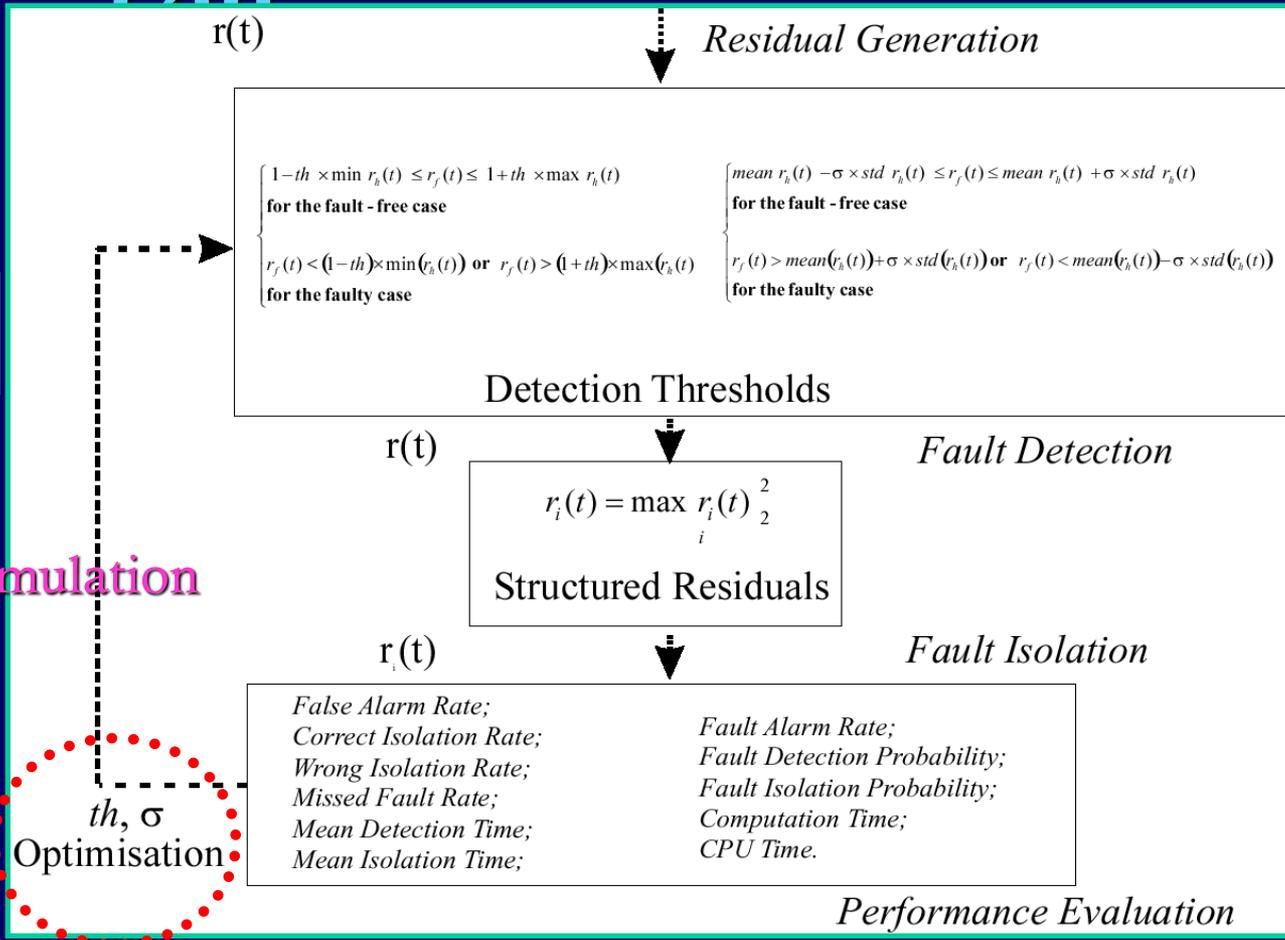


System  
Identification  
+  
Residual  
Design  
+  
Residual  
Generation

# Analytical Performance Analysis

Summary (2)

(20)



Residual  
Evaluation

+

Residual

Detection

Isolation

+

Performance  
Evaluation  
(Monte-Carlo  
analysis)

# Analytical Performance Analysis

Results (1)

Monte-Carlo

(21)

FAULT CASE	FAULT TIME	FALSE ALARM RATE	CORRECT ISOLATION RATE	WRONG ISOLATION RATE	MISSED FAULT RATE	MEAN DETECTION TIME	MEAN ISOLATION TIME	FAULT ALARM PROBABILITY	MONTE CARLO RUNS #	MEAN CPU TIME (Sec.)
Thruster2Open	100	0.000	0.840	0.160	0.000	0.100	4.814	1.000	100	0.160
Thruster2Open	300	0.000	0.940	0.060	0.000	0.100	5.417	1.000	100	0.273
Thruster2Open	500	0.000	0.980	0.020	0.000	0.100	7.885	1.000	100	0.382
Thruster2Open	750	0.000	0.930	0.070	0.000	0.100	2.099	1.000	100	0.511
Thruster2Closed	100	0.000	0.840	0.160	0.000	0.302	101.492	1.000	100	0.354
Thruster2Closed	300	0.000	0.880	0.120	0.000	3.590	107.066	1.000	100	0.366
Thruster2Closed	500	0.000	0.940	0.060	0.000	6.001	81.173	1.000	100	0.444
Thruster2Closed	750	0.000	0.610	0.390	0.030	9.737	55.509	0.970	100	0.553
Thruster4Open	100	0.000	0.790	0.210	0.000	0.100	1.846	1.000	100	0.156
Thruster4Open	300	0.000	0.970	0.030	0.000	0.100	2.665	1.000	100	0.273
Thruster4Open	500	0.000	0.970	0.030	0.000	0.100	5.587	1.000	100	0.386
Thruster4Open	750	0.000	0.860	0.140	0.000	0.104	1.997	1.000	100	0.511
Thruster4Closed	100	0.000	0.790	0.210	0.000	0.323	91.355	1.000	100	0.347
Thruster4Closed	300	0.000	0.970	0.030	0.000	2.867	104.123	1.000	100	0.367
Thruster4Closed	500	0.000	0.930	0.070	0.000	3.584	65.986	1.000	100	0.432
Thruster4Closed	750	0.000	0.600	0.400	0.000	5.676	48.422	1.000	100	0.549

$$\sigma = 2.316$$

Thruster faults

$$N_{MC} = 100$$

The Monte Carlo simulation took: 0 day(s), 2 hour(s), 6 minute(s) and 52.02 seconds.

Standard deviation parameter for false alarm rate optimisation is 2.316.

Results (1a)  
One experiment

# Analytical Performance Analysis (21a)

FAULT CASE	FAULT TIME	FAILURE CRITERION	MAX DISTURBANCE TIME	MAX DEPOINTING	MAX RATE	MIN DETECTION DELAY	FAULT DETECTION	FAULT ISOLATION	NOT ISOLABLE FROM	MAX SIMULAT TIME WIT FAULT
Thruster2Open	100	Attitude	70.500	1.926	0.751	0.100	Detectable	Not Isolable	Thruster2Closed	141.000
Thruster2Open	300	Attitude	99.950	1.965	0.728	0.100	Detectable	Isolable	None	342.000
Thruster2Open	500	Torque	99.950	1.300	0.068	0.100	Detectable	Isolable	None	526.000
Thruster2Open	750	Rate	676.500	0.240	0.058	0.100	Detectable	Isolable	None	753.000
Thruster2Closed	100	Torque	324.950	1.923	0.350	0.100	Detectable	Not Isolable	Thruster2Open	453.000
Thruster2Closed	300	Torque	609.500	1.555	0.232	65.800	Detectable	Isolable	None	619.000
Thruster2Closed	500	Torque	324.950	0.619	0.026	0.100	Detectable	Not Isolable	Thruster4Closed	590.000
Thruster2Closed	750	Torque	699.950	0.522	0.029	1.000	Detectable	Isolable	None	866.000
Thruster4Open	100	Attitude	72.500	1.963	0.636	0.100	Detectable	Isolable	None	145.000
Thruster4Open	300	Attitude	99.950	1.964	0.729	0.100	Detectable	Isolable	None	342.000
Thruster4Open	500	Torque	491.500	0.870	0.063	0.100	Detectable	Isolable	None	533.000
Thruster4Open	750	Rate	676.500	0.143	0.055	0.100	Detectable	Isolable	None	753.000
Thruster4Closed	100	Torque	464.000	0.942	0.077	0.700	Detectable	Isolable	None	478.000
Thruster4Closed	300	Torque	462.500	0.926	0.074	2.900	Detectable	Isolable	None	475.000
Thruster4Closed	500	Torque	502.500	0.790	0.041	0.100	Detectable	Not Isolable	Thruster2Closed	555.000
Thruster4Closed	750	Rate	679.000	0.188	0.038	0.300	Detectable	Isolable	None	758.000

$$\sigma = 2.316$$

Thruster faults

A single Monte-Carlo experiment

Standard deviation parameter for false alarm rate optimisation is 2.316.

# Analytical Performance Analysis

(22)

FAULT CASE	FAULT TIME	FALSE ALARM RATE	CORRECT ISOLATION RATE	WRONG ISOLATION RATE	MISSED FAULT RATE	MEAN DETECTION TIME	MEAN ISOLATION TIME	FAULT ALARM PROBABILITY	MONTE CARLO RUNS #	MEAN CPU TIME (Sec.)
Thruster2Open	100	0.000	0.880	0.120	0.000	0.100	5.056	1.000	100	0.156
Thruster2Open	300	0.000	0.940	0.060	0.000	0.100	5.753	1.000	100	0.271
Thruster2Open	500	0.000	0.930	0.070	0.000	0.100	11.276	1.000	100	0.382
Thruster2Open	750	0.000	0.950	0.050	0.000	0.100	2.342	1.000	100	0.507
Thruster2Closed	100	0.000	0.870	0.130	0.000	93.351	300.527	1.000	100	0.345
Thruster2Closed	300	0.000	0.790	0.210	0.000	75.512	181.699	1.000	100	0.368
Thruster2Closed	500	0.000	0.900	0.100	0.010	37.635	108.153	0.990	100	0.433
Thruster2Closed	750	0.000	0.900	0.100	0.030	24.251	72.109	0.970	100	0.551
Thruster4Open	100	0.000	0.810	0.190	0.000	0.100	2.380	1.000	100	0.153
Thruster4Open	300	0.000	0.860	0.140	0.000	0.100	3.594	1.000	100	0.269
Thruster4Open	500	0.000	0.980	0.020	0.000	0.100	12.379	1.000	100	0.383
Thruster4Open	750	0.000	0.930	0.070	0.000	0.133	2.384	1.000	100	0.506
Thruster4Closed	100	0.000	0.800	0.200	0.000	89.792	287.443	1.000	100	0.345
Thruster4Closed	300	0.000	0.970	0.030	0.000	79.254	176.760	1.000	100	0.362
Thruster4Closed	500	0.000	0.960	0.040	0.000	28.504	91.279	1.000	100	0.425
Thruster4Closed	750	0.000	0.960	0.040	0.000	8.352	62.459	1.000	100	0.554

$th = 0.254$

Thruster faults

$N_{MC} = 100$

The Monte Carlo simulation took: 0 day(s), 2 hour(s), 6 minute(s) and 25.70 seconds.

Maximum threshold is 1.254 and minimum threshold is 0.746.

# Analytical Performance Analysis (22a)

FAULT CASE	FAULT TIME	FAILURE CRITERION	MAX DISTURBANCE TIME	MAX DEPOINTING	MAX RATE	MIN DETECTION DELAY	FAULT DETECTION	FAULT ISOLATION	NOT ISOLABLE FROM	MAX SIMULAT TIME WIT FAULT
Thruster2Open	100	Attitude	72.500	1.842	0.666	0.100	Detectable	Isolable	None	145.000
Thruster2Open	300	Attitude	99.950	1.953	0.753	0.100	Detectable	Isolable	None	342.000
Thruster2Open	500	Torque	99.950	1.049	0.055	0.100	Detectable	Isolable	None	529.000
Thruster2Open	750	Rate	676.500	0.209	0.061	0.100	Detectable	Isolable	None	753.000
Thruster2Closed	100	Torque	99.950	0.704	0.065	0.100	Detectable	Isolable	None	455.000
Thruster2Closed	300	Torque	465.500	1.038	0.079	0.100	Detectable	Isolable	None	481.000
Thruster2Closed	500	Torque	699.950	0.586	0.044	0.100	Detectable	Isolable	None	922.000
Thruster2Closed	750	Rate	685.000	0.344	0.037	0.100	Detectable	Isolable	None	770.000
Thruster4Open	100	Attitude	71.000	1.984	0.700	0.100	Detectable	Not Isolable	Thruster4Closed	142.000
Thruster4Open	300	Attitude	99.950	1.983	0.647	0.100	Detectable	Isolable	None	344.000
Thruster4Open	500	Torque	491.000	0.856	0.054	0.100	Detectable	Isolable	None	532.000
Thruster4Open	750	Rate	676.500	0.089	0.054	0.100	Detectable	Isolable	None	753.000
Thruster4Closed	100	Torque	452.000	1.379	0.276	0.100	Detectable	Not Isolable	Thruster4Open	454.000
Thruster4Closed	300	Torque	470.500	0.819	0.040	155.100	Detectable	Isolable	None	491.000
Thruster4Closed	500	Torque	524.950	1.268	0.089	0.100	Detectable	Isolable	None	622.000
Thruster4Closed	750	Rate	524.950	0.293	0.037	0.100	Detectable	Isolable	None	756.000

$th = 0.254$

Thruster faults

Maximum threshold is 1.254 and minimum threshold is 0.746.

One single Monte-Carlo experiment

# Analytical Performance Analysis

(22)

FAULT CASE	FAULT TIME	FALSE ALARM RATE	CORRECT ISOLATION RATE	WRONG ISOLATION RATE	MISSED FAULT RATE	MEAN DETECTION TIME	MEAN ISOLATION TIME	FAULT ALARM PROBABILITY	MONTE CARLO RUNS #	MEAN CPU TIME (Sec.)
Gyro1Null	100	0.690	0.800	0.200	0.000	0.226	153.531	1.000	100	0.426
Gyro1Null	300	0.690	0.440	0.560	0.000	0.450	124.192	1.000	100	0.431
Gyro1Null	500	0.690	0.690	0.310	0.000	0.142	44.969	1.000	100	0.471
Gyro1Null	750	0.690	0.920	0.080	0.000	0.106	69.065	1.000	100	0.658
Gyro1Const	100	0.690	0.800	0.200	0.000	0.100	127.192	1.000	100	0.354
Gyro1Const	300	0.690	0.440	0.560	0.000	0.100	81.628	1.000	100	0.366
Gyro1Const	500	0.690	0.690	0.310	0.000	0.155	35.517	1.000	100	0.425
Gyro1Const	750	0.690	0.920	0.080	0.000	0.103	39.013	1.000	100	0.575
Gyro1Drift	100	0.690	1.000	0.000	0.000	0.100	209.018	1.000	100	0.683
Gyro1Drift	300	0.690	1.000	0.000	0.000	0.100	162.600	1.000	100	0.672
Gyro1Drift	500	0.690	1.000	0.000	0.000	0.192	99.912	1.000	100	0.670
Gyro1Drift	750	0.690	1.000	0.000	0.000	0.100	47.502	1.000	100	0.660

The Monte Carlo simulation took: 0 day(s), 2 hour(s), 40 minute(s) and 43.26 seconds.

Standard deviation parameter for false alarm rate optimisation is 2.237.

$\sigma = 2.237$   
Gyro faults  
 $N_{MC} = 100$

# Analytical Performance Analysis (23a)

FAULT CASE	FAULT TIME	FAILURE CRITERION	MAX DISTURBANCE TIME	MAX DEPOINTING RATE	MAX	MIN DETECTION DELAY	FAULT DETECTION	FAULT ISOLATION	NOT ISOLABLE FROM	MAX SIMULATION TIME WITH FAULT	MC CARU
Gyro1Null	100	Torque	490.000	1.389	0.141	0.100	Detectable	Isolable	None	530.000	1
Gyro1Null	300	Torque	517.000	1.364	0.123	0.300	Detectable	Isolable	None	584.000	1
Gyro1Null	500	Torque	638.000	1.515	0.169	0.100	Detectable	Not Isolable	Gyro1Const	676.000	1
Gyro1Null	750	NotDetected	699.950	0.517	0.009	0.100	Detectable	Isolable	None	1000.000	1
Gyro1Const	100	Torque	99.950	1.383	0.050	0.100	Detectable	Isolable	None	455.000	1
Gyro1Const	300	Torque	99.950	1.410	0.178	0.100	Detectable	Isolable	None	500.000	1
Gyro1Const	500	Torque	522.000	1.375	0.152	0.300	Detectable	Not Isolable	Gyro1Null	594.000	1
Gyro1Const	750	Torque	699.950	1.337	0.146	0.100	Detectable	Isolable	None	857.000	1
Gyro1Drift	100	NotDetected	699.950	0.176	0.014	0.100	Detectable	Isolable	None	1000.000	1
Gyro1Drift	300	NotDetected	699.950	0.115	0.007	0.100	Detectable	Isolable	None	1000.000	1
Gyro1Drift	500	NotDetected	699.950	0.222	0.021	0.200	Detectable	Isolable	None	1000.000	1
Gyro1Drift	750	NotDetected	699.950	0.184	0.016	0.100	Detectable	Isolable	None	1000.000	1

$\sigma = 2.237$

Gyro faults

Standard deviation parameter for false alarm rate optimisation is 2.237. One single Monte-Carlo experiment

# Analytical Performance Analysis

Results (4)  
Monte-Carlo

(24)

FAULT CASE	FAULT TIME	FALSE ALARM RATE	CORRECT ISOLATION RATE	WRONG ISOLATION RATE	MISSED FAULT RATE	MEAN DETECTION TIME	MEAN ISOLATION TIME	FAULT ALARM PROBABILITY	MONTE CARLO RUNS #	MEAN CPU TIME (Sec.)
Gyro1Null	100	0.000	0.790	0.210	0.000	10.755	264.848	1.000	100	0.407
Gyro1Null	300	0.000	0.610	0.390	0.000	24.106	185.400	1.000	100	0.428
Gyro1Null	500	0.000	0.440	0.560	0.000	0.745	90.660	1.000	100	0.473
Gyro1Null	750	0.000	0.460	0.540	0.000	1.832	147.494	1.000	100	0.653
Gyro1Const	100	0.000	0.790	0.210	0.000	0.110	144.567	1.000	100	0.347
Gyro1Const	300	0.000	0.610	0.390	0.000	0.139	89.169	1.000	100	0.357
Gyro1Const	500	0.000	0.440	0.560	0.000	0.323	58.776	1.000	100	0.419
Gyro1Const	750	0.000	0.460	0.540	0.000	0.191	60.873	1.000	100	0.573
Gyro1Drift	100	0.000	1.000	0.000	0.000	0.100	375.239	1.000	100	0.668
Gyro1Drift	300	0.000	1.000	0.000	0.000	0.301	321.583	1.000	100	0.663
Gyro1Drift	500	0.000	1.000	0.000	0.000	0.208	257.101	1.000	100	0.663
Gyro1Drift	750	0.000	1.000	0.000	0.000	0.100	126.042	1.000	100	0.657

$th = 0.3$

Gyro faults

$N_{MC} = 100$

The Monte Carlo simulation took: 0 day(s), 2 hour(s), 37 minute(s) and 22.23 seconds.

Maximum threshold is 1.300 and minimum threshold is 0.700.

Results (4a)  
One experiment

# Analytical Performance Analysis (24a)

FAULT CASE	FAULT TIME	FAILURE CRITERION	MAX DISTURBANCE TIME	MAX DEPOINTING	MAX RATE	MIN DETECTION DELAY	FAULT DETECTION	FAULT ISOLATION	NOT ISOLABLE FROM	MAX SIMULATION TIME WITH FAULT	MC CA RU
Gyro1Null	100	Torque	99.950	1.403	0.127	0.100	Detectable	Isolable	None	577.000	1
Gyro1Null	300	Torque	515.500	1.340	0.120	59.800	Detectable	Isolable	None	581.000	1
Gyro1Null	500	Torque	604.000	1.296	0.112	0.100	Detectable	Isolable	None	608.000	1
Gyro1Null	750	NotDetected	699.950	1.322	0.056	0.100	Detectable	Isolable	None	1000.000	1
Gyro1Const	100	Torque	99.950	1.403	0.054	0.100	Detectable	Isolable	None	454.000	1
Gyro1Const	300	Torque	477.000	1.409	0.169	0.100	Detectable	Isolable	None	504.000	1
Gyro1Const	500	Torque	603.000	1.382	0.148	0.100	Detectable	Isolable	None	606.000	1
Gyro1Const	750	Torque	699.950	1.340	0.150	0.100	Detectable	Isolable	None	863.000	1
Gyro1Drift	100	NotDetected	699.950	0.195	0.018	0.100	Detectable	Isolable	None	1000.000	1
Gyro1Drift	300	NotDetected	699.950	0.203	0.036	0.500	Detectable	Isolable	None	1000.000	1
Gyro1Drift	500	NotDetected	699.950	0.072	0.011	0.300	Detectable	Isolable	None	1000.000	1
Gyro1Drift	750	NotDetected	699.950	0.126	0.018	0.100	Detectable	Isolable	None	1000.000	1

$th = 0.3$

Gyro faults

A single Monte-Carlo experiment

Maximum threshold is 1.300 and minimum threshold is 0.700.

# Analytical Performance Analysis (25)

FAULT CASE	FAULT TIME	FALSE ALARM RATE	CORRECT ISOLATION RATE	WRONG ISOLATION RATE	MISSED FAULT RATE	MEAN DETECTION TIME	MEAN ISOLATION TIME	FAULT ALARM PROBABILITY	MONTÉ CARLO RUNS #	MEAN CPU TIME (Sec.)
Thruster2Open	100	0.000	0.880	0.120	0.000	0.100	4.210	1.000	100	0.161
Thruster2Open	300	0.000	0.800	0.200	0.000	0.100	5.453	1.000	100	0.275
Thruster2Open	500	0.000	0.930	0.070	0.000	0.100	8.497	1.000	100	0.387
Thruster2Open	750	0.000	0.880	0.120	0.000	0.100	2.200	1.000	100	0.515
Thruster2Closed	100	0.000	0.880	0.120	0.000	2.438	99.931	1.000	100	0.360
Thruster2Closed	300	0.000	0.710	0.290	0.000	12.619	128.392	1.000	100	0.380
Thruster2Closed	500	0.000	0.900	0.100	0.000	6.122	76.193	1.000	100	0.441
Thruster2Closed	750	0.000	0.820	0.180	0.010	0.692	43.845	0.990	100	0.549
Thruster4Open	100	0.000	0.800	0.200	0.000	0.100	1.846	1.000	100	0.158
Thruster4Open	300	0.000	0.910	0.090	0.000	0.100	2.062	1.000	100	0.275
Thruster4Open	500	0.000	0.920	0.080	0.000	0.100	9.440	1.000	100	0.390
Thruster4Open	750	0.000	0.850	0.150	0.000	0.100	2.087	1.000	100	0.514
Thruster4Closed	100	0.000	0.800	0.200	0.000	2.590	107.573	1.000	100	0.355
Thruster4Closed	300	0.000	0.970	0.030	0.000	9.932	125.861	1.000	100	0.375
Thruster4Closed	500	0.000	0.950	0.050	0.000	4.423	65.122	1.000	100	0.432
Thruster4Closed	750	0.000	0.830	0.170	0.000	0.604	47.104	1.000	100	0.552

$\sigma = 3$

Thruster faults

$N_{MC} = 100$

The Monte Carlo simulation took: 0 day(s), 2 hour(s), 9 minute(s) and 36.63 seconds.

Standard deviation parameter for false alarm rate optimisation is 3.000.

Empirical value

63

# Analytical Performance Analysis (26)

FAULT CASE	FAULT TIME	FALSE ALARM RATE	CORRECT ISOLATION RATE	WRONG ISOLATION RATE	MISSED FAULT RATE	MEAN DETECTION TIME	MEAN ISOLATION TIME	FAULT ALARM PROBABILITY	MONTE CARLO RUNS #	MEAN CPU TIME (Sec.)
Thruster2Open	100	0.140	0.910	0.090	0.000	0.100	4.170	1.000	100	0.163
Thruster2Open	300	0.140	0.960	0.040	0.000	0.100	4.801	1.000	100	0.283
Thruster2Open	500	0.140	0.760	0.240	0.000	0.100	10.953	1.000	100	0.392
Thruster2Open	750	0.140	0.930	0.070	0.000	0.100	1.691	1.000	100	0.522
Thruster2Closed	100	0.140	0.790	0.210	0.000	13.652	258.530	1.000	100	0.365
Thruster2Closed	300	0.140	0.680	0.320	0.000	83.360	169.620	1.000	100	0.379
Thruster2Closed	500	0.140	0.790	0.210	0.000	27.251	96.757	1.000	100	0.448
Thruster2Closed	750	0.140	0.950	0.050	0.000	6.097	48.418	1.000	100	0.565
Thruster4Open	100	0.140	0.700	0.300	0.000	0.100	2.168	1.000	100	0.163
Thruster4Open	300	0.140	0.700	0.300	0.000	0.100	2.166	1.000	100	0.278
Thruster4Open	500	0.140	0.990	0.010	0.000	0.100	12.938	1.000	100	0.398
Thruster4Open	750	0.140	0.990	0.010	0.000	0.100	1.759	1.000	100	0.524
Thruster4Closed	100	0.140	0.800	0.200	0.000	22.393	265.626	1.000	100	0.356
Thruster4Closed	300	0.140	0.900	0.100	0.000	70.480	172.081	1.000	100	0.377
Thruster4Closed	500	0.140	0.910	0.090	0.000	23.684	97.371	1.000	100	0.456
Thruster4Closed	750	0.140	0.950	0.050	0.000	1.448	45.238	1.000	100	0.566

The Monte Carlo simulation took: 0 day(s), 2 hour(s), 59 minute(s) and 6.62 seconds.

Maximum threshold is 1.100 and minimum threshold is 0.900.

$th = 0.1$

Thruster faults

$N_{MC} = 100$

Empirical value

64

# Analytical Performance Analysis (27)

FAULT CASE	FAULT TIME	FALSE ALARM RATE	CORRECT ISOLATION RATE	WRONG ISOLATION RATE	MISSED FAULT RATE	MEAN DETECTION TIME	MEAN ISOLATION TIME	FAULT ALARM PROBABILITY	MONTE CARLO RUNS #	MEAN CPU TIME (Sec.)
Gyro1Null	100	0.000	0.780	0.220	0.000	0.692	169.926	1.000	100	0.427
Gyro1Null	300	0.000	0.580	0.420	0.000	4.112	139.626	1.000	100	0.434
Gyro1Null	500	0.000	0.710	0.290	0.000	0.340	56.743	1.000	100	0.488
Gyro1Null	750	0.000	0.720	0.280	0.000	1.116	107.721	1.000	100	0.662
Gyro1Const	100	0.000	0.780	0.220	0.000	0.103	127.851	1.000	100	0.361
Gyro1Const	300	0.000	0.580	0.420	0.000	0.103	82.831	1.000	100	0.375
Gyro1Const	500	0.000	0.710	0.290	0.000	0.177	37.883	1.000	100	0.430
Gyro1Const	750	0.000	0.720	0.280	0.000	0.213	51.475	1.000	100	0.584
Gyro1Drift	100	0.000	1.000	0.000	0.000	0.100	323.757	1.000	100	0.688
Gyro1Drift	300	0.000	1.000	0.000	0.000	0.100	253.206	1.000	100	0.682
Gyro1Drift	500	0.000	1.000	0.000	0.000	0.299	186.594	1.000	100	0.679
Gyro1Drift	750	0.000	1.000	0.000	0.000	0.100	97.468	1.000	100	0.671

The Monte Carlo simulation took: 0 day(s), 2 hour(s), 43 minute(s) and 45.69 seconds.

Standard deviation parameter for false alarm rate optimisation is 3.000.

$$\sigma = 3$$

Gyro faults

$$N_{MC} = 100$$

Empirical value

65

# Analytical Performance Analysis (28)

FAULT CASE	FAULT TIME	FALSE ALARM RATE	CORRECT ISOLATION RATE	WRONG ISOLATION RATE	MISSED FAULT RATE	MEAN DETECTION TIME	MEAN ISOLATION TIME	FAULT ALARM PROBABILITY	MONTE CARLO RUNS #	MEAN CPU TIME (Sec.)
Gyro1Null	100	0.020	0.700	0.300	0.000	1.614	288.893	1.000	100	0.426
Gyro1Null	300	0.020	0.520	0.480	0.000	15.734	186.221	1.000	100	0.428
Gyro1Null	500	0.020	0.520	0.480	0.000	0.240	91.280	1.000	100	0.482
Gyro1Null	750	0.020	0.730	0.270	0.000	1.437	142.250	1.000	100	0.654
Gyro1Const	100	0.020	0.700	0.300	0.000	0.115	147.523	1.000	100	0.350
Gyro1Const	300	0.020	0.520	0.480	0.000	0.107	101.867	1.000	100	0.363
Gyro1Const	500	0.020	0.520	0.480	0.000	0.163	55.136	1.000	100	0.420
Gyro1Const	750	0.020	0.730	0.270	0.000	0.150	58.859	1.000	100	0.576
Gyro1Drift	100	0.020	1.000	0.000	0.000	0.100	346.956	1.000	100	0.678
Gyro1Drift	300	0.020	1.000	0.000	0.000	0.100	282.535	1.000	100	0.671
Gyro1Drift	500	0.020	1.000	0.000	0.000	0.186	242.543	1.000	100	0.665
Gyro1Drift	750	0.020	1.000	0.000	0.000	0.100	123.280	1.000	100	0.661

The Monte Carlo simulation took: 0 day(s), 2 hour(s), 44 minute(s) and 33.41 seconds.

Maximum threshold is 1.100 and minimum threshold is 0.900.

$th = 0.1$   
Gyro faults  
 $N_{MC} = 100$

# Conclusion & Further Works (1)

- ✓ Application of the residual generation technique for the MEX process.
- ✓ Linear approximated model, with sensor, actuator faults & disturbance.
- ✓ The proposed approach consists of a combined **model identification & robust FDI technique.**
- ✓ Identification approach in connection with the design of output observer schemes for FDI.
- ✓ Robustness properties against modelling errors, measurement noise, uncertainty and disturbance.

## Conclusion & Further Works (2)

- ✓ Complete design procedure for the dynamic model identification & the FDI of the *MEX process*.
- ✓ Single fault on the actuators (thruster) & the output sensors (gyro & measured quaternions) of the monitored MEX process.
- ✓ The physical knowledge of the process under observation is not be required.
- ✓ The input-output links are obtained by means of an identification scheme (ARX & EIV models) with **PEM & N4SID identification schemes**.

## Conclusion & Further Works (3)

- ✓ Results obtained by evaluating the performance achievable by Monte-Carlo simulations.
- ✓ The FDI scheme parameters were “tuned” & optimised, for obtaining good performances in terms of rate or probability optimisation.
- ✓ The achieved FDI capabilities of the proposed method indicate that the considered fault cases have be detected & isolated with arbitrary degree of false alarm rates & missed fault probabilities.

# Conclusion & Further Works (4)

- ❖ Model Identification & Validation using different sequences of data acquired for the MEX simulator.
- ❖ FDI scheme parameters ( $th$  &  $\sigma$ ) were “tuned” & validated over different data sequences.
- ❖ Adaptive thresholds?  $r(t) \equiv \Psi(t, r_f(t), u(t)) \dots$
- ❖ The achieved results indicate that the tuning of the parameters (identification & threshold values) has not to be performed during MEX flight conditions.
- The proposed approach seems to be of interest for diagnostic practical applications (of the MEX satellite).