

RESIDUAL GENERATOR COMPUTATION FOR FAULT DETECTION OF A GENERAL AVIATION AIRCRAFT

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Abstract: This paper addresses the problem of the detection and isolation of actuator faults on a general aviation aircraft, characterised by a nonlinear model, in the presence of wind gust disturbances. In particular, this work investigates the design of residual generators in order to realise complete diagnosis schemes when additive faults are present. The use of a canonical input-output polynomial description for the linearised model of the aircraft allows to compute in a straightforward way minimal order residual generators. These tools lead to dynamic filters that can guarantee both disturbance signal decoupling and robustness properties with respect to linearisation errors. The results obtained in the simulation of the faulty behaviour of a Piper PA30 are finally reported.
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1. INTRODUCTION

Actuator and sensor faults on aircraft are quite frequent, so that aircraft flight control has to exploit fault tolerant strategies in order to ensure system safety and reliability. The detection and isolation of these faults can be achieved by means of analytical redundancy schemes. Today in flight control systems, especially for the category of small commercial aircrafts, such approach is typically employed.

Analytical redundancy methods using linear or linearised models for Fault Detection and Isolation (FDI) in complex systems have received considerable attention during the last 25 years (Isermann, 1984; Gertler, 1998; Chen and Pat-

ton, 1999; Simani *et al.*, 2002; Frank, 1990; Basseville and Nikiforov, 1993). In recent years a great deal of works have been carried on to the synthesis of residual generators with geometric approach (Massoumnia, 1986; Chen and Patton, 1999; Balas and Bokor, 2000). It is worth observing that Balas and Bokor have developed the project of robust detection filters by means of an LPV approach.

This work investigates the residual generator computation on the basis of a linearised model of a multivariable nonlinear system with additive faults and disturbances, by following the minimal polynomial approach suggested in (Frisk and Nyberg, 2001). The system under diagnosis is modelled in terms of input-output polynomial description, so that the residual generation problem can

be reduced to the determination of the null-space of a specific polynomial matrix associated to the process model. In particular, the use of canonical input-output forms allows to compute in a straightforward fashion an analytical expression for the basis of such a null-space and upper and lower bounds for the order of the dynamic residual generator. The proposed FDI approach has been applied to a nonlinear model of a Piper PA30. The residual generators have been designed on the basis of linearised models in different flight conditions and experimented with the data from nonlinear model flight simulator in Matlab/Simulink environment.

2. AIRCRAFT MATHEMATICAL DESCRIPTION

In order to adopt the linear FDI technique that will be presented in the following, a nonlinear model of the aircraft is used for computing trim values and linearised models corresponding to different flight conditions.

The mathematical description of the PA30 is a classical nonlinear six degree of freedom aircraft model (rigid body) whose motion occur as a consequence of applied forces and moments (aerodynamic, thrust and gravitational). The parameters in the analytic representation of the aerodynamic actions have been obtained from wind tunnel experimental data of a Piper PA30, as reported e.g. in (Koziol, 1971), and the aerodynamic actions are expressed along the axes of the wind reference system. The nonlinear attitude model is given by following relations (using nomenclature of Table (2)):

$$\begin{aligned}\dot{V} &= F_x \frac{\cos \alpha \cos \beta}{m} + F_y \frac{\sin \beta}{m} + F_z \frac{\sin \alpha \cos \beta}{m} \\ \dot{\alpha} &= \frac{-F_x \sin \alpha + F_z \cos \alpha}{mV \cos \beta} + Q - (P \cos \alpha + R \sin \alpha) \tan \beta \\ \dot{\beta} &= \frac{-F_x \cos \alpha \sin \beta + F_y \cos \beta - F_z \sin \alpha \sin \beta}{mV} + P \sin \alpha + \\ &\quad - R \cos \alpha \\ \dot{P} &= \frac{M_x I_z + M_z I_{xz} + PQ I_{xz} (I_x - I_y + I_z)}{I_x I_z - I_{xz}^2} + \\ &\quad + \frac{QR (I_y I_z - I_{xz}^2 - I_z^2)}{I_x I_z - I_{xz}^2} \\ \dot{Q} &= \frac{M_y + PR (I_z - I_x) - P^2 I_{xz} + R^2 I_{xz}}{I_y} \\ \dot{R} &= \frac{M_x I_{xz} + M_z I_x + PQ (I_x^2 - I_x I_y + I_{xz}^2)}{I_x I_z - I_{xz}^2} + \\ &\quad + \frac{QR I_{xz} (-I_x + I_y - I_z)}{I_x I_z - I_{xz}^2} \\ \dot{\phi} &= P + Q \sin \phi \tan \theta + R \cos \phi \tan \theta \\ \dot{\theta} &= Q \cos \phi - R \sin \phi\end{aligned}$$

Moreover, the model has been completed by means of a first order dynamic model of a 4-pistons aspirated engine with the throttle aperture as input and the thrust intensity as output. The atmosphere model consists of a stochastic mathematical description of gusts which are modelled air velocity components along body axes w_u , w_v and w_w . Their correlation times and variances are $\tau_u = 2.326[s]$, $\tau_v = 7.143[s]$, $\tau_w = 0.943[s]$, $E[w_u^2] = E[w_v^2] = E[w_w^2] = 0.7[(m/s)^2]$ respectively. It is worth observing that in the PA30 Matlab/Simulink model the servo-actuator description of elevator, aileron rudder and throttle have been included.

The linearised model used for FDI purposes embed the linearisation of the attitude model, of the engine and of the guide variables H and ψ so that it can be written as follows

$$\dot{x}(t) = Ax(t) + Bc(t) + Ed(t) \quad (1)$$

with

$$\begin{aligned}x(t) &= [\Delta V \ \Delta \alpha \ \Delta \beta \ \Delta P \ \Delta Q \ \Delta R \ \dots \\ &\quad \Delta \phi \ \Delta \theta \ \Delta \psi \ \Delta H \ \Delta n]^T \\ c(t) &= [\Delta \delta_e \ \Delta \delta_a \ \Delta \delta_r \ \Delta \delta_{th}]^T \\ d(t) &= [w_u \ w_v \ w_w]^T\end{aligned} \quad (2)$$

where Δ denotes the variations of the considered variables and $c(t)$ and $d(t)$ are the control inputs and the disturbances, respectively. The output equation corresponding to model (1) is of the type $y(t) = Cx(t)$ where the rows of C correspond to rows of the identity matrix, depending on the measured variables.

3. RESIDUAL GENERATOR FUNCTIONS FOR FDI

Let us consider a linear, time-invariant, continuous-time system described by the following input-output equation

$$P(s)y(t) = Q(s)u(t) \quad (3)$$

where s is the derivative operator and $P(s)$ and $Q(s)$ are polynomial matrices with dimension $(m \times m)$ and $(m \times \ell)$ respectively, with $P(s)$ non-singular. The terms $u(t)$ and $y(t)$ are, respectively, the ℓ -dimensional and m -dimensional input and output vectors of the considered multivariable system. Models of type (3) can be frequently found in practice by applying well-known physical laws to describe the input-output dynamical links of various systems and are a powerful tool in all fields where the knowledge of the system state

Table 1. Nomenclature

V	True Air Speed (TAS)	δ_e	elevator deflection angle
α	angle of attack	δ_a	aileron deflection angle
β	angle of sideslip	δ_r	rudder deflection angle
P	roll rate	δ_{th}	throttle aperture percentage
Q	pitch rate	X, Y	horizontal coordinates (inertial reference system)
R	yaw rate	H	altitude (inertial reference system)
ϕ	bank angle	γ	flight path angle
θ	elevation angle	m	airplane mass
ψ	heading angle		
n	engine shaft angular rate		
$\begin{bmatrix} I_x & 0 & -I_{xz} \\ 0 & I_y & 0 \\ -I_{xz} & 0 & I_z \end{bmatrix}$			airplane inertia moments matrix
F_x, F_y, F_z			total force components along body axes
M_x, M_y, M_z			total moment components along body axes

does not play a direct role, such as residual generator design, identification, decoupling, output controllability, etc. Algorithms to transform state-space models to equivalent input-output polynomial representations and vice versa are available (Beghelli and Guidorzi, 1976).

A constructive proof of the existence and uniqueness of a canonical form for a given pair $\{P(s), Q(s)\}$ can be found in (Beghelli and Guidorzi, 1976). In the same work an efficient and simple algorithm for transforming a generic polynomial representation to the equivalent canonical one is also described.

Remark 1. Note that the integers $\nu_i = \deg \tilde{p}_{ii}(s)$ ($i = 1, \dots, m$) equal the corresponding row-degrees. Because of the canonical form, matrix $\tilde{P}(s)$ is row-reduced, *i.e.*

$$\tilde{P}(s) = D(s)M + L(s), \quad (4)$$

where $D(s) = \text{diag}[s^{\nu_1}, s^{\nu_2}, \dots, s^{\nu_m}]$ and the highest-row-degree coefficient matrix M is non singular since it is a triangular matrix with ones along the main diagonal. Moreover the canonical representation $\{\tilde{P}(s), \tilde{Q}(s)\}$ leads directly to a correspondent canonical state-space realization

$$\begin{aligned} \dot{x}(t) &= \tilde{A}x(t) + \tilde{B}u(t) \\ y(t) &= \tilde{C}x(t) + \tilde{D}u(t). \end{aligned} \quad (5)$$

The integers ν_i are the ordered set of Kronecker invariants associated to the pair $\{\tilde{A}, \tilde{C}\}$ of every observable realization of $\{P(s), Q(s)\}$.

In order to design residual generators of minimal order, model (3) can be firstly transformed into its canonical representation $\{\tilde{P}(s), \tilde{Q}(s)\}$; this step can be omitted if the minimal order constraint is relaxed. Then, matrix $\tilde{Q}(s)$ can be decomposed according to the following structure

$$\tilde{P}(s)y(t) = [\tilde{Q}_c(s) \ \tilde{Q}_d(s) \ \tilde{Q}_f(s)] \begin{bmatrix} c(t) \\ d(t) \\ f(t) \end{bmatrix} \quad (6)$$

where $c(t)$ is the ℓ_c -dimensional known-input vector, $d(t)$ is the ℓ_d -dimensional disturbance vector, $f(t)$ is the ℓ_f -dimensional monitored fault vector and $\ell_c + \ell_d + \ell_f = \ell$.

Remark 2. Eq. (6) considers also the cases of additive faults $f_c(t)$ on the input and output sensors. In particular when additive faults $f_c(t)$ on the input are considered, the input vector measurements can be written as

$$c^*(t) = c(t) + f_c(t) \quad (7)$$

and Eq. (6) with $f(t) = 0$, becomes $\tilde{P}(z)y(t) = \tilde{Q}_c(s)c^*(t) + \tilde{Q}_d(s)d(t) - \tilde{Q}_c(s)f_c(t)$. Analogously, when additive faults $f_o(t)$ on the output sensors of the system are considered the output vector measurements can be written as

$$y^*(t) = y(t) + f_o(t). \quad (8)$$

In this case, it results that $\tilde{P}(s)y^*(t) = \tilde{Q}_c(s)c(t) + \tilde{Q}_d(s)d(t) + \tilde{P}(s)f_o(t)$.

A general linear residual generator for the fault detection process of system (6) is a filter of type

$$R(s)r(t) = S_y(s)y(t) + S_c(s)c(t). \quad (9)$$

System (9) processes the known input-output data and generates the residual $r(t)$, *i.e.* a signal which is “small” (ideally zero) in the fault-free case and is “large” when a fault is acting on the system. Without loss of generality, $r(t)$ can be assumed to be a scalar signal. In such condition $R(s)$ is a polynomial with degree greater than or equal to the row-degree of $S_c(s)$ and $S_y(s)$,

in order to guarantee the physical realisability of the filter. Moreover, if $R(s)$ has all roots in the left half complex plane, filter (9) is asymptotically stable. An important aspect of the design concerns the decoupling of the disturbance $d(t)$ in order to produce a correct diagnosis in all operating conditions. Equation (6) can be rewritten in the form

$$\tilde{P}(s)y(t) - \tilde{Q}_c(s)c(t) + \tilde{Q}_f(s)f(t) = \tilde{Q}_d(s)d(t). \quad (10)$$

Premultiplying all the terms in (10) by a row polynomial vector $L(s)$ belonging to the left null-space of $\tilde{Q}_d(s)$, $\mathcal{N}_\ell(\tilde{Q}_d(s))$, we obtain

$$L(s)\tilde{P}(s)y(t) - L(s)\tilde{Q}_c(s)c(t) - L(s)\tilde{Q}_f(s)f(t) = 0. \quad (11)$$

Starting from Eq. (11) with $f(t) = 0$, it is possible to obtain a residual generator of type (9) by setting:

$$\begin{aligned} S_y(s) &= L(s)\tilde{P}(s) \\ S_c(s) &= -L(s)\tilde{Q}_c(s) \\ R(s) &= (s+p_1)(s+p_2)\dots(s+p_{n_f}) = \\ &= s^{n_f} + a_1s^{n_f-1} + \dots + a_{n_f}, \end{aligned} \quad (12)$$

where n_f is the maximal row-degree of the pair $\{L(s)\tilde{P}(s), L(s)\tilde{Q}_c(s)\}$. The polynomial $R(s)$ can be arbitrarily selected. The choice $R(s) = (s+p_1)(s+p_2)\dots(s+p_{n_f})$ leads to an asymptotically stable filter when the real parts of the n_f poles $p_i (i = 1, 2, \dots, m)$ are negative. In this way, in absence of fault, equation (9) can be rewritten also in the form

$$R(s)r(t) = L(s)\tilde{P}(s)y(t) - L(s)\tilde{Q}_c(s)c(t) = 0 \quad (13)$$

When a fault is acting on the system the residual generator is governed by the relation

$$R(s)r(t) = -L(s)\tilde{Q}_f(s)f(t) \quad (14)$$

and $r(t)$ assumes values that are different from zero if $L(s)$ does not belong to the $\mathcal{N}_\ell(\tilde{Q}_f(s))$. In these conditions the design freedom in the choice of the matrix $L(s)$ can be used to optimise the sensitivity properties of $r(t)$ to the fault $f(t)$, for example by maximising the steady-state gain of the transfer function $L(s)\tilde{Q}_f(s)$. Another design choice regards the location of the roots of the polynomial $R(s)$ in the left-half s -plane, which influences the frequency response of the residual generator and, consequently, its robustness with respect to input-output measurement noises, modelling errors, parameter uncertainties, etc. In other words the diagnostic capabilities of a residual generator strongly depend on an accurate selection of the terms $L(s)$ and $R(s)$. In order to determine all possible residual generators of minimal

order it is necessary to compute a minimal basis of $\mathcal{N}_\ell(\tilde{Q}_d(s))$. Under the assumption that matrix $\tilde{Q}_d(s)$ is of full normal rank, *i.e.* $\text{rank } \tilde{Q}_d(s) = \ell_d$, $\mathcal{N}_\ell(\tilde{Q}_d(s))$ has dimension $m - \ell_d$ and a minimal basis of it can be computed easily. It can be noted that in absence of disturbances $\ell_d = 0$, so that $\mathcal{N}_\ell(\tilde{Q}_d(s))$ coincides with the whole vector space. Consequently, a set of residual generators for system (6) with $f(t) = 0$ can be expressed as

$$R_{ri}(s)r_i(t) = \tilde{P}_{ri}(s)y(t) - \tilde{Q}_{c_{ri}}(s)c(t) \quad (i = 1, 2, \dots, m) \quad (15)$$

where $\tilde{P}_{ri}(s)$ and $\tilde{Q}_{c_{ri}}(s)$ are the i -th rows of matrices $\tilde{P}(s)$ and $\tilde{Q}_c(s)$, respectively, ν_i is the degree of $\tilde{P}_{ri}(s)$ and $R_{ri}(s)$ is an arbitrary polynomial with degree equal to ν_i and with all the roots with negative real part. Since $\tilde{Q}_{c_{ri}}(s)$ cannot show a degree greater than ν_i , the physical realisability of the residual generator is guaranteed. In general, for $0 < \ell_d < m$ matrix $\tilde{Q}_d(s)$ can be partitioned in the following way

$$\tilde{Q}_d(s) = \begin{bmatrix} \tilde{Q}_{d1}(s) \\ \tilde{Q}_{d2}(s) \end{bmatrix}, \quad (16)$$

where matrices $\tilde{Q}_{d1}(s)$ and $\tilde{Q}_{d2}(s)$ have dimension $\ell_d \times \ell_d$ and $(m - \ell_d) \times \ell_d$ respectively. It can be assumed, without loss of generality, that matrix $\tilde{Q}_{d1}(s)$ is non singular. In this case it can be easily verified that a basis of $\mathcal{N}_\ell(\tilde{Q}_d(s))$ is given by the polynomial matrix

$$B(s) = [\tilde{Q}_{d2}(s) \text{adj } \tilde{Q}_{d1}(s) \dots -\det \tilde{Q}_{d1}(s) I_{m-\ell_d}] \quad (17)$$

by assuming $\text{adj } \tilde{Q}_{d1}(s) = 1$ for $\ell_d = 1$. By partitioning $\tilde{P}(s)$ and $\tilde{Q}_c(s)$ as $\tilde{Q}_d(s)$ in (16)

$$\tilde{P}(s) = \begin{bmatrix} \tilde{P}_1(s) \\ \tilde{P}_2(s) \end{bmatrix} \quad \tilde{Q}_c(s) = \begin{bmatrix} \tilde{Q}_{c1}(s) \\ \tilde{Q}_{c2}(s) \end{bmatrix} \quad (18)$$

a basis of the residual generators (9) for the system (6) with $f(t) = 0$ is obtained by replacing in relation (12) the row polynomial vector $L(s)$ with the polynomial matrix $B(s)$, *i.e.*

$$\begin{aligned} S_y(s) &= \tilde{Q}_{d2}(s) \text{adj } \tilde{Q}_{d1}(s) \tilde{P}_1(s) + \\ &\quad -\det \tilde{Q}_{d1}(s) \tilde{P}_2(s) \\ S_c(s) &= -\tilde{Q}_{d2}(s) \text{adj } \tilde{Q}_{d1}(s) \tilde{Q}_{c1}(s) + \\ &\quad +\det \tilde{Q}_{d1}(s) \tilde{Q}_{c2}(s) \\ R(s) &= \text{diag} [R_1(s) R_2(s) \dots R_{m-\ell_d}(s)], \end{aligned} \quad (19)$$

where the degree of the polynomial $R_i(s)$ is n_{fi} ($i = 1, \dots, m - \ell_d$), that is the degree of the i -th row of the matrix $S_y(s)$. By denoting with n_f^* the minimal value of the integers n_{fi} ($i = 1, \dots, m - \ell_d$) the following theorem can be stated.

Theorem 1. The order n_f^* of a minimal order residual generator for the system (6) is constrained in the following range

$$\nu_{\min} \leq n_f^* \leq (\ell_d + 1) \nu_{\max} \quad (20)$$

where ν_{\min} and ν_{\max} are the least and the greatest Kronecker invariant respectively.

The lower bound can be obtained in the no-disturbance case ($\ell_d = 0$) from relation (15) by selecting the row of $\tilde{P}(s)$ associated to the least Kronecker invariant. The upper bound can be obtained by taking into account the maximal degree of the polynomials of the matrices in (19).

4. APPLICATION EXAMPLE

To show the advantages brought by the application of the proposed fault detection and isolation scheme to the the general aviation PIPER PA-30 aircraft model addressed in Section 2, some numerical results obtained in the Matlab/Simulink environment are reported.

The computation of disturbance decoupling residual generators described in Section 3 for actuator FDI has been performed by considering the linearised model for the aircraft presented in Section 2. Such linearised model of the aircraft corresponds to a flight condition branch of a complete trajectory described by:

- radius of curvature 1000[m]
- speed $V = 50[\frac{m}{s}]$
- altitude $H = 330[m]$
- flap = 0° .

The residual generator filters are therefore fed by the 4 input – 8 output data acquired from the continuous time, time invariant, nonlinear dynamic aircraft Matlab/Simulink model described in Section 2.

In particular, a bank of 4 residual generator filters have been used to detect actuator faults regarding the 4 input control variables $c(t) = [\Delta\delta_e, \Delta\delta_a, \Delta\delta_r, \Delta\delta_{th}]^T$.

The *actuator faults* $f_a(t)$ are modelled in the following way:

$$c(t) = c_h(t) + f_a(t) \quad (21)$$

where $c_h(t)$ are the fault-free inputs that feed the residual generators and $f_a(t)$ are step functions. In this situation, Eq. (6) becomes $\tilde{P}(s)y(t) = \tilde{Q}_c(s)c_h(t) + \tilde{Q}_d(s)d(t) + \tilde{Q}_c(s)f_a(t)$. Moreover, in order to obtain fault *isolation* properties, each residual generator function of the considered bank is fed by all but one the 4 control input

signals and by the 8 output variables $y(t) = [\Delta V, \Delta Q, \Delta\theta, \Delta H, \Delta P, \Delta R, \Delta\phi, \Delta\psi]^T$.

Hence, each filter of the bank is independent of one of the 4 input signals and then is also insensitive to the corresponding fault signals. Obviously, the residual generator bank has been designed to be decoupled from 3 wind gust signals $d(t) = [w_u, w_v, w_w]^T$, that represent disturbance terms acting on the aircraft system.

The capabilities of the fault detection and isolation system are hence related to the properties of the residual generator functions in the presence of disturbance and nonlinearity that cannot be decoupled. In particular, according to Section 3, the synthesis of the filters for FDI has been performed by choosing a linear combination of residual generator $\{S_c(s), S_y(s)\}$ functions that maximise the steady-state gain of the transfer matrix between fault signals $f_a(t)$ and residual functions $r(t)$. Moreover, for each residual generator, the poles connected to the polynomial $R(s)$ have been chosen in order to optimise fault isolation properties of the filter bank, *i.e.* by minimising linearisation error effects on the residual functions.

As an example, the 4 residual functions generated by the filter bank under both fault-free and faulty conditions are shown in Figures (1) and (2), respectively. Continuous lines represent the fault-

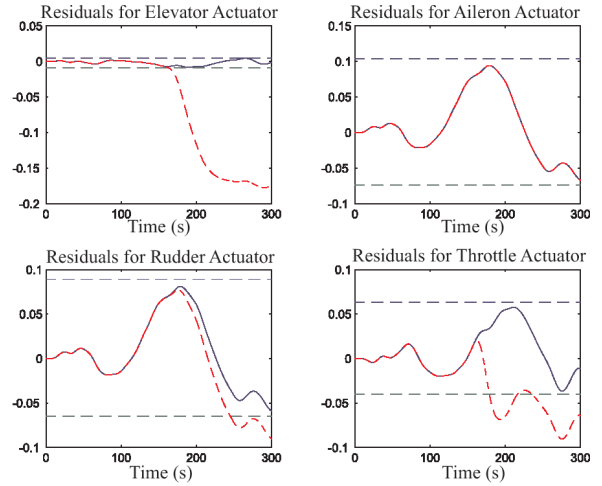


Fig. 1. Residuals of the bank for the isolation of the aileron actuator fault

free residual functions, while the dotted lines depict the faulty residual signals. The faults have been added to the aileron (Figure (1)) and to the rudder (Figure (2)) actuator signals of the considered aircraft, commencing at time $t = 150s$.

It is worth noting that the second residual function of Figure (1), for the isolation of a fault regarding the aileron, does not depend on a fault affecting aileron itself, as the corresponding residual filter has been designed to be insensitive to

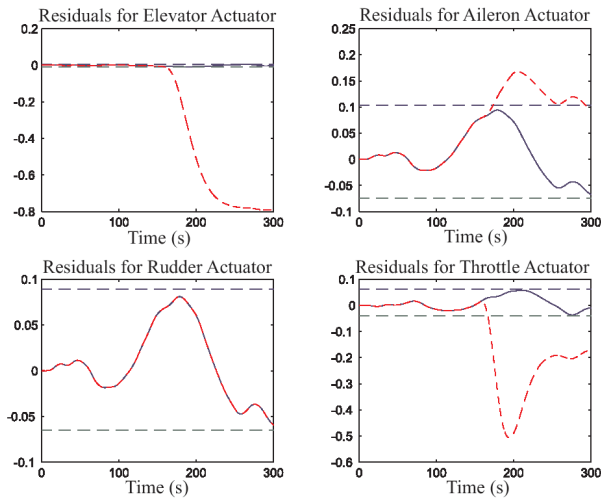


Fig. 2. Residuals of the bank for the isolation of the rudder actuator fault.

that input signal. On the other hand, Figure (2) depicts the third residual function for the isolation of a fault regarding the rudder. This function is independent of a fault affecting rudder signal, as the related filter has been designed to be decoupled from that input signal.

In order to determine the range out of which the fault is detectable, the maximum and minimum values assumed by $r(t)$ in fault-free conditions must be computed with an acceptable false-alarms rate. Table (2) collects the minimal detectable fault amplitudes on the actuators, when $r(t)$ is monitored in order to perform the isolation of the considered fault cases. The minimal

Table 2. Minimal detectable actuator faults.

Actuator variable	Fault Size
Elevator deflection angle	0.28°
Aileron deflection angle	0.7°
Rudder deflection angle	4.5°
Throttle aperture %	8%

detectable fault values in Table (2) are expressed in the unit of measure of the actuator signals and are relative to the case in which the occurrence of a fault must be detected and isolated as soon as possible.

Finally, the performance of the residual generators seems to assess the diagnostic capabilities of the suggested technique. Moreover, the proposed strategy for the FDI on the actuators appear to be promising for diagnostic application to general aviation aircrafts.

5. CONCLUSIONS

In this paper a residual generation technique for a general aviation aircraft, described by a nonlinear

model, with additive actuator faults and disturbance signals has been analysed. The proposed approach has shown robustness properties against linearisation errors and disturbance decoupling. The use of a canonical input-output polynomial representation for a linearised model of the considered nonlinear dynamic model leads to a simple computation of the minimal order residual generators in polynomial form. Minimal detectable fault sizes are comparable those achievable with other classical FDI schemes. The robustness properties of the suggested approach with respect to both measurement noises and parameter variations of the system require further investigations.

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