RAPM-Based Selective Preventive Maintenance to Improve Availability for Series-Parallel Systems

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Abstract: In a series-parallel system, the diverse degradation and failure trajectories of its components call for tailored maintenance strategies to extend the overall sustainability of the system. To improve the availability of series-parallel systems under the premise of ensuring safety, a Reliability Allocation-based Programming Model (RAPM) is proposed in this paper. First, the reliability of components is allocated based on weights. Then, to satisfy strict safety requirements, the RAPM is modeled under reliability constraints. Minimizing maintenance costs is the first goal, while reducing the component reliability gap is the second. The proposed RAPM provides information on the state trends of components in the series-parallel system. The case study focuses on the traction converter systems of electric locomotives. Experimentally validated with actual data, the result is to provide a predicted schedule for updating maintenance interventions during planned downtimes, potentially yielding substantial economic benefits for railway companies.

Keywords: planned downtimes, reliability allocation, selective preventive maintenance, availability, series-parallel systems.

1. INTRODUCTION

Systems in industries and the military must complete tasks without much downtime. For example, rail and aviation systems can only be maintained between tasks. Doing the right maintenance during these breaks boosts system performance and success in later tasks (Levitin et al. (2023)). By performing appropriate maintenance activities during planned downtimes, system performance can be significantly improved, thereby greatly increasing the probability of success for subsequent tasks. However, maintenance resources (such as budget, duration, and manpower) are often limited, and not all ideal maintenance activities can be carried out during planned downtimes. In such cases, it is necessary to select the optimal subset of feasible maintenance activities, to be executed in advance during planned downtimes, ensuring the maximum probability of success for subsequent tasks. This maintenance strategy is referred to as selective preventive maintenance (Tambe (2022)).

Cassady et al. (2001) proposed four improvements to the preventive maintenance enumeration method first proposed by Rice et al. (1998), and summarized the maintenance optimization problem as a nonlinear knapsack problem. Samrout et al. (2005) modified the genetic algorithms of Bris et al. (2003) and Tsai et al. (2001), introducing an ant colony optimization algorithm for minimizing preventive maintenance costs in series-parallel systems. Khatab et al. (2016) considered the duration and rest time of the tasks to be random. To improve the reliability of the system after selective preventive maintenance, Zhong et al. (2019) established a fuzzy multi-objective nonlinear chance-constrained programming model based on reliability and cost for preventive maintenance of wind farms in the offshore wind energy field. Zhu et al. (2021) considered the system reliability, maintenance cost and uncertain system profit within the remaining service life. In recent work, Rudek and Rudek (2024), the total time for rolling stock maintenance is reduced by scheduling transportation tasks, and the maintenance problem is modeled as a job scheduling problem to maximize the availability of rolling stock under preventive maintenance. However, few scholars take component reliability into the process of maintenance decision-making when performing maintenance optimization. Advanced methods in system reliability allocation, as demonstrated in references Zhong et al. (2023) and Gholinezhad (2024), have been effectively employed in the initial phases of system design and manufacturing. These methodologies predominantly address complex systems configured in series and parallel. The incongruent degradation and failure trends of individual components during the operational maintenance phase of series-parallel systems remain a critical knowledge gap. Therefore, understanding the influence of system reliability and its correlation with component reliability on maintenance decision-making emerges as a pivotal research concern.

With the rapid development of the transportation industry, ensuring the safety of equipment during tasks is of paramount importance. To prevent unplanned downtime (referring to task interruptions caused by equipment failures), certain critical systems are often configured in a redundant manner, such as the energy supply system in electric locomotives (Tolbert et al. (2024)). According to the real statistics of electric locomotives, the failure rate of the traction converter system accounts for 85.44% of the total failure rate. This is the main cause of train delays. More than half of these failures are caused by IGBT (Insulated Gate Bipolar Transistor) failures.

This paper investigates the problem of selective preventive decision-making in systems with series-parallel redundancy, a Reliability Allocation-based Programming Model (RAPM) is proposed. To meet the required reliability level for the system's next task and minimize unplanned downtime during task execution, maintenance activities are performed on system components during planned downtime. Taking the traction converter system of electric locomotives as an example, Weibull distribution is fitted to component lifetimes. The minimum reliability requirements for the system are determined, and component reliability is allocated based on weights. The weight coefficients are related to component MTBF (Mean Time Between Failures) and system reliability requirements. The minimum maintenance cost is the first objective, and the minimum component reliability gap is the second objective based on the allocated reliability requirements. Decisions on updating components are solved under the constraint of meeting the reliability of the system. The final result is a predictive schedule for complete update interventions during the system's operational period. The experimental validation is conducted using real data from the traction converter system of electric locomotives.

The compositional features and maintenance strategies of the studied system are discussed in Section 2. The construction process of the proposed RAPM and the corresponding solution algorithm are given in Section 3. Section 4 examines a practical data case, offering descriptions and experimental discussions on three performance indicators. Section 5 concludes and outlines future research directions.

2. TRACTION CONVERTER SYSTEMS OF ELECTRIC LOCOMOTIVES

2.1 Structure

The electric locomotive has been developed to meet the high-power AC traction requirements of railway passenger and freight transport. Each vehicle is equipped with an independent and identical AC-DC-AC traction drive system. Electric locomotives obtain electrical energy from the catenary through the pantograph. Subsequently, two identical onboard traction step-down transformers converter the voltage to an output voltage of 970 V to the two traction converter subsystems, which comprise two four-quadrant rectifiers and two inverters. The parallel four-quadrant rectifiers jointly perform rectification and voltage stabilization, which is then input to the two parallel inverters. Each inverter drives an electric motor, achieving separate control of each axis of the motor, as shown in Figure 1. The advantage of this parallel design is that the functional supply of the locomotive has 75% redundancy.

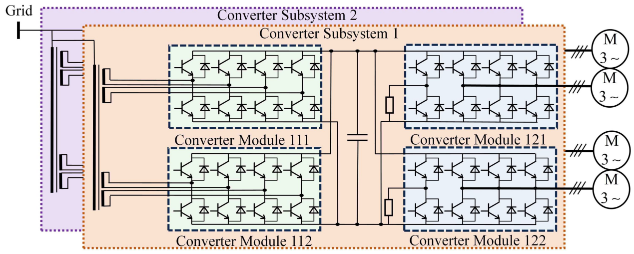

Fig. 1. Topological structure and composition of the traction converters systems

In practice, a common strategy to enhance power handling is to integrate eight IGBT components into a unified power module. Both the four-quadrant rectifier and inverter are designed in a modular fashion, utilizing eight IGBT components, making them interchangeable converter modules. The notable advantage of this design is that it avoids the complex troubleshooting process associated with the failure of individual IGBT components. There is no longer a need for frequent installation of single IGBT components. Instead, the entire module can be directly substituted when any IGBT component within the converter module malfunctions or fails.

2.2 Maintenance Strategies

In electric locomotive maintenance, downtime is categorized as planned or unplanned. Planned downtime includes routine preventive maintenance scheduled by distance or time, as shown in Figure 2. Unplanned downtime occurs when an electric locomotive unexpectedly halts due to malfunctions, requiring swift fault maintenance for a quick response and repair. This is a major contributor to train delays and can result in considerable economic losses if extended.

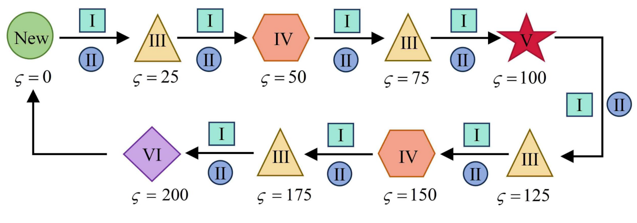

Fig. 2. The preventive maintenance procedures for electric locomotives

The preventive maintenance for the electric locomotives is categorized into six levels, namely Class I to Class VI. Where Class V and Class VI belong to advanced overhaul, and the others are operation and maintenance. In particular, Class VI maintenance involves a comprehensive disassembly and thorough overhaul of the locomotive to meet the standards of a new electric locomotive. When "$\varsigma = 0$", the travel distance of the electric locomotive is zero, indicating a new locomotive. When "$\varsigma = 25$", it signifies a travel distance of 25×10^4 Km, meeting the requirements for the first inspection of Class III. After an additional 25×10^4 Km, totaling 500,000 kilometers, the locomotive will require its first inspection for Class IV. With a cumulative travel distance of 75×10^4 Km, the locomotive undergoes its second Class III maintenance. Between two Class III maintenances, the locomotive undergoes a Class I maintenance every 6.25 $\times10^4$ Km and a Class II maintenance every 12.5 $\times10^4$ Km. Importantly, higher-level maintenance tasks (maintenance content) encompass all lower-level repairs.

In the traction converter systems of electric locomotives, almost all failures occur during the operation of the locomotive. Each malfunction of the converter module results in unscheduled downtime, requiring maintenance personnel to perform repairs based on the actual fault conditions. Class I maintenance typically involves fault diagnosis through the self-check system of the locomotive, while maintenance at other levels requires inspection and repair of critical components. Therefore, preventive maintenance on the traction converter system should be scheduled during planned downtimes from Class II to Class VI. However, an elevated probability of module replacements inevitably results in increased maintenance costs. Therefore, effective maintenance of converter modules represents a complex decision-making process.

3. RAPM-BASED SELECTIVE PREVENTIVE MAINTENANCE

3.1 Weighted-Based Reliability Allocation

Due to the stochastic nature of converter module failures, there is no clear relationship between the fatigue lifetime and failures of converter modules. In theory, during the extended operational fatigue process of the converter modules, failures can occur at any time. The life distribution can be characterized by the Weibull distribution (Weibull (2021)).

The failure probability density function of the Weibull distribution is

$$N_{\chi_{ijk}}(t) = \frac{\beta_{ijk}}{\alpha_{ijk}}\left(\frac{t}{\alpha_{ijk}}\right)^{\beta_{ijk}-1} exp\left(-\left(\frac{t}{\alpha_{ijk}}\right)^{\beta_{ijk}}\right) \quad (1)$$

The corresponding cumulative failure distribution function is

$$F_{ijk}(t) = \int_0^t \mathrm{N}_{\chi_{ijk}}(x)\,dx = 1 - exp\left(-\left(\tfrac{t}{\alpha_{ijk}}\right)^{\beta_{ijk}}\right) \quad (2)$$

where i is the converter subsystem sequence number, j is the converter unit category, "$j = 1$" represents the rectifier unit, "$j = 2$" represents the inverter unit, k is the converter module sequence number in the converter unit category, χ_{ijk} represents the converter module in the unit of category j in subsystem i. Correspondingly, α_{ijk} is the dimension parameter and β_{ijk} is a shape parameter.

Under the failure mode of the Weibull distribution, reliability is defined as the probability of a system functioning normally within a specified time period, which is expressed by $R_{ijk}(t)$. The function of reliability $R_{ijk}(t)$ is

$$R_{ijk}(t) = 1 - F_{ijk}(t) = exp\left(-\left(\frac{t}{\alpha_{ijk}}\right)^{\beta_{ijk}}\right) \quad (3)$$

In reliability analysis, the rectifier and inverter units are treated as a parallel system. The converter subsystem can be regarded as a series system. The entire traction converter system represents a complex system where two converter subsystems operate in parallel, as depicted in Figure 3.

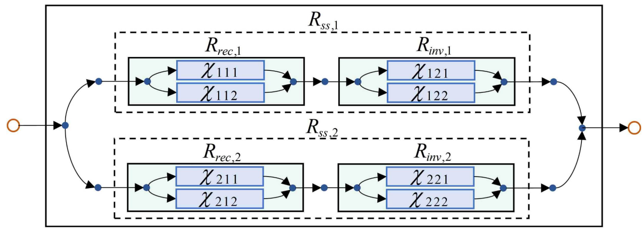

Fig. 3. Reliability block diagram of the traction converter systems

The reliability of the rectifier unit and the inverter unit in the subsystem i are

$$R_{rec,i}(t) = 1 - \prod_{k=1}^{2}(1 - R_{i1k}(t)) \quad (4)$$

$$R_{inv,i}(t) = 1 - \prod_{k=1}^{2}(1 - R_{i2k}(t)) \quad (5)$$

The reliability of the converter subsystem i is

$$R_{ss,i}(t) = R_{rec,i}(t)\,R_{inv,i}(t) \quad (6)$$

The reliability of the entire traction converter system is a function of the reliability of the converter modules, which is expressed by $\varphi(R_{ijk}(t))$ and calculated by the simultaneous formula (4), formula (5), and formula (6). The result is

$$\begin{aligned} R_s(t) &= \varphi(R_{ijk}(t)) \\ &= 1 - \prod_{i=1}^{2}\left(1 - \prod_{j=1}^{2}\left(1 - \prod_{k=1}^{2}(1 - R_{ijk}(t))\right)\right) \end{aligned} \quad (7)$$

According to engineering specifications and operational realities, the reliability requirement of the traction converter system in electric locomotives is denoted by R_s^*. The reliability requirements allocated to each converter module are represented by R_{ijk}^* and are expressed as

$$R_{ijk}^* = \omega_{ijk} R_s^* \quad (8)$$

The coefficient ω_{ijk} is dynamic, which is contingent on the fluctuating MTBF. This coefficient is represented as the multiplication of a parameter C_{ijk}, which fluctuates with MTBF, and an intrinsic parameter ξ linked to the reliability requirement of system, there is

$$\omega_{ijk} = C_{ijk}\xi \quad (9)$$

where the coefficient C_{ijk} can be calculated as

$$C_{ijk} = \frac{\mathrm{MTBF}_{ijk}}{\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\mathrm{MTBF}_{ijk}} \quad (10)$$

3.2 RAPM-Based Maintenance Decision-Making

In the decision-making process for converter module replacement, the tri-dimensional variable $H^{(\varsigma)}$ is determined by the assignment of the binary vector $\vartheta_{ijk}^{(\varsigma)} \in \{0,1\}$ and is expressed as

$$H^{(\varsigma)} = \left\{ \begin{matrix} \left\{\vartheta_{111}^{(\varsigma)}, \vartheta_{112}^{(\varsigma)}\right\} & \left\{\vartheta_{121}^{(\varsigma)}, \vartheta_{122}^{(\varsigma)}\right\} \\ \left\{\vartheta_{211}^{(\varsigma)}, \vartheta_{212}^{(\varsigma)}\right\} & \left\{\vartheta_{221}^{(\varsigma)}, \vartheta_{222}^{(\varsigma)}\right\} \end{matrix} \right\} \tag{11}$$

where, ς is the distance traveled during planned downtimes from Class II to Class VI, $\vartheta_{ijk}^{(\varsigma)} = 1$ represents the module retention and $\vartheta_{ijk}^{(\varsigma)} = 0$ represents the module replacement.

The constraint of the proposed RAPM is to ensure that the reliability of the system meets the prescribed requirements. The foremost objective is to minimize the number of module replacements. A secondary goal is to minimize the gap between the reliability of converter modules and the required reliability, particularly for modules that fall short of meeting the established standards. These objectives are formulated as follows:

$$z1 = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\left(1 - \vartheta_{ijk}^{(\varsigma)}\right) \tag{12}$$

$$z2 = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\left(R_{ijk}^{*} - R_{ijk}\left(\vartheta_{ijk}^{(\varsigma)}\varsigma\right)\right) \tag{13}$$

In the planned downtime period of electric locomotives, i.e., when $t = \varsigma$, there is

$$\begin{aligned} &\min_{\vartheta_{ijk}^{(\varsigma)}} \; P_1 z1 + P_2 z2 \\ &s.t. \\ &R_s\left(\vartheta_{ijk}^{(\varsigma)}\varsigma\right) = \varphi\left(R_{ijk}\left(\vartheta_{ijk}^{(\varsigma)}\varsigma\right)\right) \\ &= 1 - \prod_{i=1}^{2}\left(1 - \prod_{j=1}^{2}\left(1 - \prod_{k=1}^{2}\left(1 - R_{ijk}\left(\vartheta_{ijk}^{(\varsigma)}\varsigma\right)\right)\right)\right) \\ &\geq R_s^{*} \end{aligned} \tag{14}$$

where P_1 stands as the primary position and P_2 assumes the secondary position under the condition $P_1 \gg P_2$.

In the process of finding the optimal solution for the proposed RAPM, the iteration unfolds, commencing with newly introduced electric locomotives and concluding with Class VI maintenance during planned downtime. The iterative step size corresponds to the distance between neighboring maintenance levels. This entire process is succinctly described in Algorithm 1.

Initial tasks of the algorithm involve configuring replacement states for converter modules and determining parameters like the distance for planned outage. We then solve for each scheduled maintenance level, updating the replacement status of the module with the greatest reliability deficit if the system reliability doesn't meet standards. This iterative process continues until system reliability meets requirements at each maintenance level during planned downtimes. The outcome includes the replacement status of converter modules for every planned outage during the operational period.

Algorithm 1 The algorithm to solve RAPM

Input: Reliability $R_{ijk}(t)$, reliability requirements R_{ijk}^{*} and R_s^{*}, set of distances $U = \{0, 12.5, 25, \cdots, 200\}$, set $i \in \{1,2\}$, $j \in \{1,2\}$ and $k \in \{1,2\}$
Output: $H^{(\varsigma)}$ for all $\varsigma \in U$

```
H^(ς) = {ϑ_ijk^(ς)} = 1; //all i, j, k and ς;
ς = 0; R̂_ijk(t) = R_ijk(t);
while ς<200 do
  ς = ς + 12.5;
  while φ(R̂_ijk(t)) <R_s^* do
    //Function φ base on formula (7);
    ΔR_ijk = R_s^* − R̂_ij(ς);
    [m, n, q] =find(ΔR_ijk == max{R_ijk});
    ϑ_mnq^(ς−12.5) = 0;
    R̂_mnq(ς + t) = R_mnq(t); //Update reliability.
  end while
end while
```

4. CASE STUDY

4.1 Dataset Description

The dataset used in this study is from a locomotive operating company, which details a comprehensive record of faults encountered by the electric locomotive for a whole year. The dataset comprises detailed maintenance information for all unplanned interruptions during the locomotive's routine operations, encompassing various malfunction types, corresponding remedial measures, and the specific distances covered by the locomotive at the time of each fault occurrence. This study focuses on the failure of the traction converter systems of electric locomotives during unplanned outage maintenance.

4.2 Simulation Setup

Statistical assessment of faults in all converter modules of electric locomotives in the dataset is conducted. Fitting the lifetimes of the converter modules with the Weibull distribution, as shown in Table 1, is the basis for reliability analysis and preventive maintenance optimization simulation.

Table 1. The parameters of Weibull distribution for converter modules

Converter Module	MTBF $\left(\times 10^4 Km\right)$	Dimension Parameter	Shape Parameter
χ_{111}	90.12	96.96	3.90
χ_{112}	86.34	95.05	3.55
χ_{121}	77.14	85.42	2.15
χ_{122}	78.70	88.86	2.98
χ_{211}	86.47	94.66	4.73
χ_{212}	89.16	95.25	4.20
χ_{221}	55.31	57.87	1.21
χ_{222}	71.57	79.50	3.51

4.3 Performance Indicators

(1) ***Availability*** is commonly employed to reflect the fault and repair characteristics of a system, which is expressed by A. High availability signifies that

the system remains operable and functional for a significant duration. In this paper, availability is defined as the ratio of the actual distance traveled by an electric locomotive to the equivalent total distance, can that be obtained

$$A = \frac{DT}{DT + DL_{plan} + DL_{unplan}} \tag{15}$$

where DT is the actual distance traveled, DL_{plan} is the distance of loss from the planned downtimes, and DL_{unplan} is the distance of loss from unplanned downtimes.

(2) ***Average Replacement Rate*** reflects the mean level of failure or replacement of system components. A lower average replacement rate indicates higher maintenance efficiency and lower maintenance costs for the system. The average replacement rate is determined by the mean replacement probability of converter modules in this paper.

(3) ***Fault Percentage*** refers to the relative proportion of a certain fault in the system. In this paper, it refers to the proportion of the number of electric locomotive delays caused by the failure of the traction converter systems to the total number of delays, which is used to quantify the impact of the fault of the traction converter systems on the task timeliness.

4.4 Results

Figure 4 presents the fitting results of the Weibull lifetime distribution for eight converter modules in the dataset. From the results, it is evident that these converter modules exhibit significant variations in Weibull lifetime distribution, influenced by factors such as module installation position, functionality, and operational conditions. Furthermore, the Weibull lifetime distribution profile of χ_{221} exhibits a notable distinction from that of the remaining modules. This dissimilarity could be attributed to the limited quantity of statistical data and the circumstance that a majority of electric locomotives have not covered substantial distances when logged as malfunctioning.

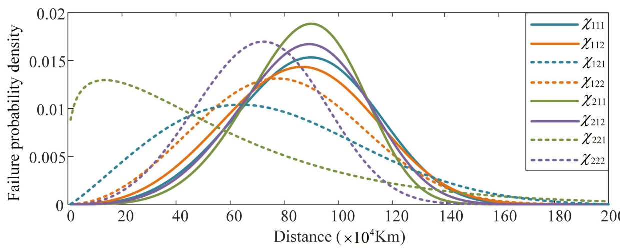

Fig. 4. The Weibull distribution of converter modules

The reliability requirements for converter modules are allocated, which is presented in Table 2. The reliability requirement of the traction converter system is specified as 99.5%. According to the results, the shorter the MTBF of the converter module, the lower its reliability requirement. In reliability allocation, modules that are more prone to failures typically have lower reliability requirements. This allocation method balances the overall system reliability and reduces the module replacement rate to lower maintenance costs. However, for actual traction converter systems in electric locomotives, considerations should also be given to the working environment and real usage conditions of the converter modules for a more accurate assessment. This is assumed that each converter module has the same working environment and service conditions in this paper.

Table 2. The results of reliability allocation

χ_{111}	χ_{112}	χ_{121}	χ_{122}	χ_{211}	χ_{212}	χ_{221}	χ_{222}
93.53	89.62	80.07	81.69	89.75	92.54	57.40	74.28

Figure 5 provides a detailed description of the results of the proposed RAPM in this study, including its content and implementation timeline. Throughout the operational life of the electric locomotive, converter modules of the inverter unit undergo complete replacement before or during the initial Class IV maintenance, with 2 to 3 replacement occurrences before the system-wide update. In contrast, converter modules of the rectifier unit are primarily replaced during the initial Class V maintenance, with a lower replacement frequency compared to the inverter unit. The method proposed in this study successfully maintains the reliability of traction converter systems at 99.6% or above, as detailed in Figure 6.

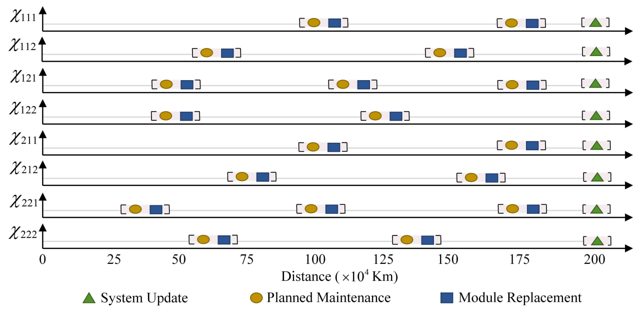

Fig. 5. The predictive schedule for intervention updates

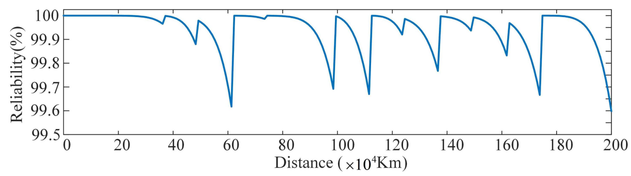

Fig. 6. The reliability of traction converter systems under selective preventive maintenance

To validate the effectiveness of the proposed method in this paper, a comparative experiment was conducted, and the results are shown in Table 3. The experiment involved the replacement of all converter modules during each Class III maintenance period. Compared with the actual operation, the available rate of the proposed method is increased by 24%. The availability rate under the same module average replacement rate is 1.8 times that of the comparison experiment. The results indicate that replacing converter modules during planned maintenance significantly improves the availability and reduces the rate of delays caused by faults during operation. In particular, a balance between reliability and maintenance cost can be achieved by purposefully optimizing the maintenance strategy.

Table 3. The comparison of performance indicators of experimental

	Minimum Reliability (%)	Availability (%)	Average Replacement Rate (%)	Fault Percentage (%)
Actual Operation	/	96.23	15	85.44
The Class III as Replacement Cycle	99.46	96.61	301	57.44
Method of This Paper	**99.62**	**96.46**	**105**	**65.82**

5. CONCLUSION

This paper investigates the problem of selective preventive decision-making in systems with series-parallel redundancy. The primary objective is to ensure the system achieves the necessary reliability to perform upcoming tasks with minimal interruptions and delays. Consequently, this requires selective component updates during planned downtimes. Using the traction converter system of electric locomotives as an example, component lifetimes are modeled with the Weibull distribution. Considering both component fatigue cycles and system reliability requirements simultaneously, a weight-based allocation of least reliability is performed. Finally, the RAPM is constructed with dual objectives of minimizing maintenance costs and minimizing the reliability gap between components. The predicted schedule for intervention updates during the service period is solved. The method is validated through experimentation and discussion using a real-world data case.

We are currently conducting in-depth research on technical solutions to ensure system safety while maximizing economic value. This requires a quantitative assessment of maintenance costs and losses from task interruptions. Accurate prediction of component failures is also imperative. The crucial trade-offs can be achieved not only by deciding which component to maintain and during which downtime but also by determining the optimal time within the system's operational timeframe to perform maintenance on that component.

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