Finite-frequency fuzzy fault-tolerant static output feedback H_∞ control for Diesel engine air-path system

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Abstract: This paper focuses on the design of H_∞ finite-frequency (FF) fault-tolerant static output feedback (FTSOFC) problem of Diesel engine air-path system with consideration of external disturbances and actuator faults. Initially, a Diesel engine air-path nonlinear model is described by Takagi-Sugeno (T-S) fuzzy model based descriptor approach. The aim is to regulate intake and exhaust manifold pressures to the desired reference pressures by controlling the Geometry Turbine (VGT) and Exhaust Gas Recirculation (EGR) valves. Then, the robust integrator-based control strategy is developed to track the desired reference signals despite the presence of disturbances and actuator faults. By using the extended Generalized Kalman Yakubovich Popov (GKYP) lemma, Lyapunov functions and independent slack matrices, sufficient conditions are established to ensure both the good tracking of reference pressures and the prescribed H_∞ performance with FF domain of the fuel flow variation and actuator faults. Finally, simulation results are given to demonstrate the effectiveness of the proposed approach.

Keywords: Fault-tolerant, Finite frequency, Diesel engine, static output feedback control, T-S fuzzy systems

1. INTRODUCTION

Diesel engines are still supported by motorists and transport companies, thanks to their performance, which is generally superior to that of gasoline engines. Nevertheless, particulate emissions such as nitrogen oxides (NOx) and particles hazardous to human health remain the main concerns. Diesel engines have to meet three expectations: control costs, improved reliability and ave a low impact on the environment. To meet these requirements, technologies such as variable geometry turbochargers (VGT) and exhaust gas recirculation (EGR) systems have been introduced. The main idea is to recirculate a quantity of burnt gases into the cylinder and inject fuel at high pressure to improve combustion. A large number of theoretical results have demonstrated a significant reduction in pollutant emissions Stefanopoulou et al. (2000); Cheng et al. (2020).

Static output feedback controller has also received considerable attention among control issues because it can easily be implemented in practice and is much simpler than dynamic output feedback control. Recently, some important results by LMI approach El-Amrani et al. (2022, 2023); Elias et al. (2022). Faults, particularly actuator Fault are frequently encountered in many practical systems, often resulting in poor performance or even instability for system dynamics. It is therefore necessary and important to design a fault-tolerant controller against the actuator faults. In the past decade, many great results on Fault-tolerant controller systems have been reported in Tu et al. (2023); Selvaraj et al. (2023).

To the best of our knowledge, no work has been reported on the modeling and control of the Diesel engine air-path system for FF domain with actuator faults. Thus, in this paper, we study the problem of H_∞ FF FTSOFC design based on T-S fuzzy model. Diesel engine Air-path nonlinear model is firstly described by TS fuzzy model based descriptor approach. Then, a FF FTSOFC guaranteeing a good reference pressures tracking with a H_∞ performance is designed with consideration of external disturbances and actuator faults. By constructing the extended GKYP lemma, Lyapunov functions and independent slack matrices, sufficient conditions are established to ensure both the reference signal tracking and H_∞ performance with FF domain of the fuel flow variation and actuator faults. The theoretical results are given in the form of LMI.

2. PROBLEM FORMULATION AND PRELIMINARIES

In this work, let's consider the Diesel engine air-path structure of a variable geometry turbocharged Diesel engine equipped with an EGR system governed by a valve is shown in fig. 1. The engine system comprises: the four-cylinder engine, exhaust and intake manifolds, the VGT, the EGR valve and cooler, turbocharger. The Diesel en-

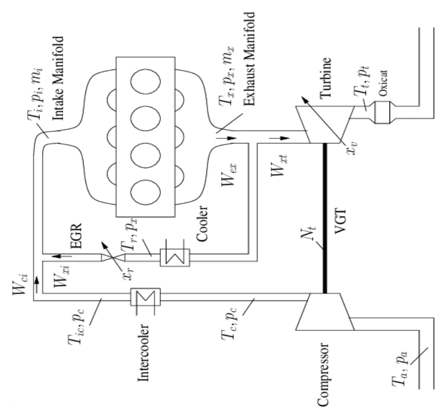

Fig. 1. Air-path model of Diesel engine.

gine air-path model with the following form Stefanopoulou et al. (2000)

$$\begin{aligned} \dot{P}_i &= T_i(W_{ci} - W_{ie} + W_{xi}) \times \frac{R}{V_{in}}; \\ \dot{P}_x &= T_x(W_{ei} + m_f - W_{xi} - W_{xt}) \times \frac{R}{V_{ex}}; \\ \dot{P}_c &= (-P_c + \eta_m p_t) \times \frac{1}{\tau} \end{aligned} \tag{1}$$

where states P_i, P_x, P_c represent, respectively, the intake manifold pressure, exhaust manifold pressure, compressor power.
Compressor mass flow rate W_{ci} is related to the compressor power by

$$W_{ci} = \eta_c P_c \times \frac{1}{T_a c_p[(-1 + \frac{P_i}{p_a})^{\mu}]} \tag{2}$$

Engine intake mass flow rate W_{ie} is calculated by the speed density equation

$$W_{ie} = \frac{1}{120} \times \frac{\eta_v N V_d P_i}{RT_i} \tag{3}$$

The flow through the EGR valve W_{xi} is established as follows

$$W_{xi} = \begin{cases} \frac{\kappa_0}{\sqrt{RT_x}}\left(\alpha^{\frac{1}{2}}\left[\frac{2}{\alpha+1}\right]^{\kappa_1}\right); \frac{P_i}{P_x} \le \left[\frac{2}{\alpha+1}\right]^{\kappa_1} \\ \frac{\kappa_0}{\sqrt{RT_x}}\sqrt{2\kappa_3}\left((\frac{P_i}{P_x})^{\frac{2}{\alpha}} - (\frac{P_i}{P_x})^{\kappa_2}\right); \frac{P_i}{P_x} > \left[\frac{2}{\alpha+1}\right]^{\kappa_1} \end{cases}$$

where

$$\kappa_0 = A_{egr}(\vartheta_{egr})P_x;\ \kappa_1 = \frac{\alpha+1}{2(\alpha-1)};\ \kappa_2 = \frac{\alpha+1}{\alpha};\ \kappa_3 = \frac{\alpha}{\alpha-1}$$

and $A_{egr}(\vartheta_{egr})$ is the effective area of the valves as a function of $\vartheta_{egr} \in [0, 100]$ % being the valve opening positions.
Turbine flow W_{xt} is given by

$$W_{xt} = \left[c(\frac{P_x}{p_a} - 1) + d\right]\frac{P_x}{p_a}\sqrt{\frac{T_a}{T_x}}\sqrt{\frac{2p_a}{P_x}(1 - \frac{p_a}{P_x})})A_{vgt}(\vartheta_{vgt}) \tag{4}$$

with the VGT opening position $\vartheta_{vgt} \in [0, 1]$ can be calculated inversely from equation (4) once the turbine flow rate is specified by the controller.
The turbine power p_t is modeled by

$$p_t = W_{xt} c_p T_x \tau_t (1 - (\frac{p_a}{P_x})^{\mu}) \tag{5}$$

Using equations (1-5), the nonlinear model of the air-path of a diesel engine can be rewritten as:

$$\begin{aligned} \dot{P}_i &= -\frac{\eta_v V_d N}{120V_i}P_i + \frac{R\eta_c T_i}{V_i c_p T_a((\frac{P_i}{p_a})^{\mu} - 1)}P_c + \frac{RT_i}{V_i}W_{xi}; \\ \dot{P}_x &= \frac{\eta_v V_d N T_x}{120 V_x T_i}P_i - \frac{RT_x}{V_x}(W_{xi} + W_{xt} - m_f); \\ \dot{P}_c &= \frac{-1}{\tau}P_c + \frac{c_p \eta_t T_x}{\tau}(1 - (\frac{p_a}{P_x})^{\mu})W_{xt}; \end{aligned} \tag{6}$$

Define $x_1 = P_i$, $x_2 = P_x$ and $x_3 = P_c$ are the state variables; $u_1 = W_{xi}$, $u_2 = W_{xt}$ are the control inputs; $w = m_f$ denote the disturbance input. Then, the state-space form of the Diesel engine Air-path model can be given as :

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Dw(t) \\ y(t) &= Lx(t) \end{aligned} \tag{7}$$

Where

$$A = \begin{pmatrix} -\frac{\eta_v V_d N}{120V_i} & 0 & \frac{R\eta_c T_i}{V_i c_p T_a}\xi_1(x_1); \\ \frac{\eta_v V_d N}{120V_x}\frac{T_x}{T_i} & 0 & 0 \\ 0 & 0 & \frac{-1}{\tau} \end{pmatrix};\ x = \begin{pmatrix} P_i \\ P_x \\ P_c \end{pmatrix};$$

$$B = \begin{pmatrix} \frac{RT_i}{V_i} & 0 \\ -\frac{RT_x}{V_x} & -\frac{RT_x}{V_x} \\ 0 & \frac{c_p\eta_t T_x}{\tau}\xi_2(x_2) \end{pmatrix};D = \begin{pmatrix} 0 \\ \frac{RT_x}{V_x} \\ 0 \end{pmatrix};u = \begin{pmatrix} W_{xi} \\ W_{xt} \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix};\ \xi_1(x_1) = \frac{1}{(\frac{P_i}{p_a})^{\mu} - 1};\ \xi_2(x_2) = 1 - (\frac{p_a}{P_x})^{\mu} \tag{8}$$

One way to take into account the nonlinearities of the model is to use a polytopic approach such as the Takagi-Sugeno model Takagi et al. (1985). The fuzzy model is described by fuzzy IF-THEN rules, the set of which represents the approximation of the nonlinear system. A fuzzy T-S model has the following form : **Plant Rule *i*:** IF $\xi_1(t)$ is N_1^i and,..., and $\xi_s(t)$ is N_s^i THEN

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t) + Dw(t) \\ y(t) &= Lx(t) \end{aligned} \tag{9}$$

Where $\xi_1(t)$,..., $\xi_s(t)$ are premise variables; $N_j^i(i = 1, ..., q; j = 1, ..., s)$ are fuzzy sets; q is the number of **IF-THEN** rules; s is the number of premise variables and A_i, B_i, D, L_i are known matrices with appropriate dimensions.
These functions are bounded by $\underline{\xi}_1 \le \xi_1 \le \overline{\xi}_1$ and $\underline{\xi}_2 \le \xi_2 \le \overline{\xi}_2$, for $(P_{i_{min}} \le P_i \le P_{i_{max}})$ and $(P_{x_{min}} \le P_x \le P_{x_{max}})$ with

$$\begin{aligned} \underline{\xi}_1 &= \frac{1}{-1 + (\frac{\overline{P}_i}{p_a})^{\mu}};\ \overline{\xi}_1 = \frac{1}{-1 + (\frac{\underline{P}_i}{p_a})^{\mu}}; \\ \underline{\xi}_2 &= 1 - (\frac{p_a}{\overline{P}_x})^{\mu};\ \overline{\xi}_2 = 1 - (\frac{p_a}{\underline{P}_x})^{\mu} \end{aligned} \tag{10}$$

from this decomposition, the follow functions are given

$$N_{11} = \frac{\xi_1 - \underline{\xi}_1}{\overline{\xi}_1 - \underline{\xi}_1};\ N_{12} = 1 - N_{11};\ N_{21} = \frac{\xi_2 - \underline{\xi}_2}{\overline{\xi}_2 - \underline{\xi}_1};\ N_{22} = 1 - N_{21} \tag{11}$$

where $N_{lk}(\xi_l(t))$ is the grade of membership of $\xi_l(t)$ in N_{lk}, with $l, k = 1, 2$. This leads four local models by combining four functions

$$M_i(\xi(t)) = N_{lk}(\xi_l(t))N_{kl}(\xi_k(t)) \tag{12}$$

with $l = 1:2,\ k = 1:2$, and $i = 1:4$.
Finally $h_i(\xi(t))$ is seen as the normalized weight of each IF-THEN rules, given by

$$h_i(\xi(t)) = \frac{M_i(\xi(t))}{\sum_{i=1}^{4} M_i(\xi(t))} \tag{13}$$

We assume

$$\sum_{i=1}^{4} M_i(\xi(t)) = 1; \quad 0 \leq M_i(\xi(t)) \tag{14}$$

for any $\xi(t)$. Therefore, for all t we have

$$\sum_{i=1}^{4} h_i(\xi(t)) = 1; \quad 0 \leq h_i(\xi(t)) \tag{15}$$

The overall T-S fuzzy model is inferred as follows

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{4} h_i(\xi(t))\{A_i x(t) + B_i u(t)\} + Dw(t); \\ y(t) &= Lx(t) \end{aligned} \tag{16}$$

For simplicity, introduce the following notations

$$h_i = h_i(\xi(t); \quad A(h) = \sum_{i=1}^{4} h_i A_i; \quad B(h) = \sum_{i=1}^{4} h_i B_i$$

Then, the T-S fuzzy model (16) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= A(h)x(t) + B(h)u(t) + Dw(t); \\ y(t) &= Lx(t) \end{aligned} \tag{17}$$

The energy of the disturbance $w(t)$ is assumed to be dominated in a known rectangular FF region Σ given as follows

$$\Sigma = w \in \mathbb{R} | \ |w| \leq \bar{w}_l; \quad \bar{w}_l > 0 \tag{18}$$

For initial conditions, $x_0 = (70KPa; 55KPa; 500\ Watts)^T$, fig. 2 a comparison of the intake and exhaust manifold pressures responses obtained by the two models (T-S model and nonlinear model). Based on the results of this figure, it is assumed that the satisfaction of the T-S model representation is approved. Now, formulate the FTSOFC problem, the actuator fault given in Yang et al. (2001) is adopted in this work, for the control input $u(t)$, we denote $u_f(t)$ to describe the signal sent from the actuator, and satisfies

$$u_f(t) = Fu(t) \tag{19}$$

where, the actuator fault matrix F is defined as F =diag$\{F_1, F_2\}$, $0 \leq \underline{F}_j \leq F_j \leq \overline{F}_j \leq 1$; $(j = 1, 2)$, with $\underline{F}_j$ and $\overline{F}_j$ being known real constants. We denote $\overline{F}$ =diag$\{\overline{F}_1, \overline{F}_2\}$; $\underline{F}$ =diag$\{\underline{F}_1, \underline{F}_2\}$. Then, the actuator fault function matrix F can be represented by

$$F = \delta_1 + \Omega\delta_2; \quad |\Omega| \leq I \tag{20}$$

where $\delta_1 = \frac{\overline{F}+\underline{F}}{2}$, $\delta_2 = \frac{\overline{F}-\underline{F}}{2}$, Ω =diag$\{\zeta_1, \zeta_2, \zeta_3\}$. In this paper, the FF FTSOFC objective is to regulate the intake and exhaust manifold pressures of the Diesel engine, to ensure tracking of given output reference y_r, we introduce the integrator of the tracking errors as follows :

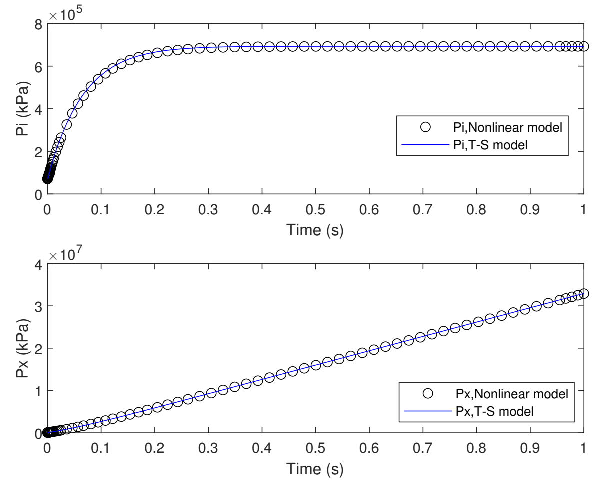

Fig. 2. Intake and Exhaust manifold pressures of the nonlinear and the T-S model

$$\dot{e}(t) = y_r(t) - y(t) \tag{21}$$

proposes the following fuzzy FTSOFC :

$$u(t) = K_1(h)\int (y_r(t) - y(t))dt + K_2(h)y(t) \tag{22}$$

and

$$K_1(h) = \sum_{i=1}^{4} h_i K_{1i}; \quad K_2(h) = \sum_{i=1}^{4} h_i K_{2i}$$

Then

$$u_f(t) = \begin{pmatrix} 0 & FK_1(h) & FK_2(h) \end{pmatrix} \begin{pmatrix} x(t) \\ y(t) \\ e(t) \end{pmatrix} \tag{23}$$

Let $\eta(t) = \begin{pmatrix} x^T(t) & y^T(t) & e^T(t) \end{pmatrix}^T$ and Combining fuzzy system (17) and tracking error (21), then the following closed-loop fuzzy system:

$$\begin{aligned} \bar{E}\dot{\eta}(t) &= \bar{A}(h)\eta(t) + \bar{D}\bar{\omega}(t); \\ r(t) &= \bar{L}\eta(t) \end{aligned} \tag{24}$$

where

$$\bar{E} = \begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \bar{A}(h) = \begin{pmatrix} A(h) & B(h)FK_1(h) & B(h)FK_2(h) \\ 0 & 0 & -I \\ -L & 0 & I \end{pmatrix};$$

$$\bar{L} = \begin{pmatrix} 0 & I & 0 \end{pmatrix}; \quad \bar{D} = \begin{pmatrix} D & 0 \\ 0 & I \\ 0 & 0 \end{pmatrix}; \quad \bar{w}(t) = \begin{pmatrix} w(t) \\ y_r(t) \end{pmatrix} \tag{25}$$

The problem of FF fuzzy H_∞ FTSOFC consists in presenting a controller such that the closed-loop fuzzy system (24) is said with H_∞ performance bound $\gamma > 0$ in the presence of actuator faults, if it is asymptotically stable, moreover, the following inequality holds under the zero initial condition

$$\int_{w\in\Sigma} Y^T(w)Y(w)dw \leq \int_{w\in\Sigma} W^T(w)W(w)dw \tag{26}$$

where $Y(w)$, $W(w)$ are the Fourier transform of the exogenous disturbance $\bar{w}(t)$ and the measured output $r(t)$, respectively.

3. FINITE FREQUENCY H_∞ FT SOFC DESIGN USING DESCRIPTOR SYSTEMS

To deal with actuator fault and external disturbances, the following lemmas are needed to demonstrate the main results.

Lemma 1. Gahinet et al. (1994) Given matrices Γ, Λ and Φ. There exist a matrix $\mathcal{X}$ such that the following statements are equivalent :

(i) $\Phi + \Gamma\mathcal{X}\Lambda^T + \Lambda\mathcal{X}^T\Gamma^T < 0$
(ii) $\Gamma^\perp\Phi\Gamma^{\perp T} < 0;\ \Lambda^\perp\Phi\Lambda^{\perp T} < 0$

Lemma 2. Tuan et al. (2014) If the following equations are satisfied :

$$\Delta_{ii} < 0, \quad 1 \le i \le r;$$
$$\frac{1}{r-1}\Delta_{ii} + \frac{1}{2}(\Delta_{ij} + \Delta_{ji}) < 0; \quad 1 \le i \ne j \le r;$$

Then the following parameterized linear matrix inequality holds:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))\Delta_{ij} < 0$$

Lemma 3. Chadli et al. (2017) The T-S fuzzy descriptor system (17) is admissible if and only if the following statements are equivalent :

(i) The exist a matrix Q such that :

$$\bar{A}^TQ + Q^T\bar{A} < 0; \quad \bar{E}^TQ = Q^T\bar{E} \ge 0 \tag{27}$$

(ii) There exists matrices $Q > 0$ and S such that :

$$(Q\bar{E} + SR)^T\bar{A} + \bar{A}^T(Q\bar{E} + SR) < 0 \tag{28}$$

with $\bar{E}^TR = 0$ and $R \in \mathbb{R}^{n\times(n-n_{\bar{E}})}$.

Lemma 4. El-Amrani et al., 2023 Assume that the closed-loop system (24) is asymptotically stable, with FF H_∞ performance index γ, if there exists symmetric matrices $P(h)$, $U(h) = U^T(h) > 0$ such that

$$\begin{bmatrix} \bar{A}(h) & \bar{D} \\ \bar{E} & 0 \end{bmatrix}^T \begin{bmatrix} -U(h) & P(h) \\ P(h) & \bar{\omega}_l^2U(h) \end{bmatrix} \begin{bmatrix} \bar{A}(h) & \bar{D} \\ \bar{E} & 0 \end{bmatrix} + \begin{bmatrix} \bar{L}^T\bar{L} & 0 \\ 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{29}$$

Remark 1. If all the parameters in Lemma 4 is independent of h, T-S fuzzy system becomes a linear system, Lemma 4 reduces to the GKYP lemma Iwasaki et al. (2005).

Theorem 1. For unknown actuator fault matrix F, closed-loop fuzzy system (24) is asymptotically stable and H_∞ performance index γ of FF domain Σ, if there exist symmetric matrices $U(h)$, $P(h) > 0$ and $Q(h) > 0$ and matrices $G(h)$, S satisfying the following conditions :

$$\begin{bmatrix} -U(h) - G(h) - G^T(h) & \Gamma_1 & G(h)\bar{D} & 0 \\ \star & \Gamma_2 & G(h)\bar{D} & \bar{L}^T \\ \star & \star & -\gamma^2 I & 0 \\ \star & \star & \star & -I \end{bmatrix} < 0 \tag{30}$$

$$\begin{bmatrix} -G(h)^T - G(h) & -G(h) + \Gamma_3 \\ \star & G(h)\bar{A}(h) + \bar{A}^T(h)G^T(h)] \end{bmatrix} < 0 \tag{31}$$

where

$$\Gamma_1 = U(h)\bar{E} + G(h)\bar{A}(h) - G^T(h);$$
$$\Gamma_2 = \bar{\omega}_l^2\bar{E}^TU(h)\bar{E} + G(h)\bar{A}(h) + \bar{A}^T(h)G^T(h);$$
$$\Gamma_3 = (Q(h)\bar{E} + RS)^T + \bar{A}^T(h)G^T(h);$$
$$R = \begin{bmatrix} R_1^T & R_2^T & R_3^T \end{bmatrix}^T$$

and $\bar{E}^TR = 0$ and $R \in \mathbb{R}^{n\times(n-n_{\bar{E}})}$.

Proof 1. First, show that (29) is equivalent to (30), consider that (29) can be rewritten as follows :

$$\begin{bmatrix} \bar{A}(h) & \bar{D} \\ I & 0 \\ 0 & I \end{bmatrix}^T \Phi \begin{bmatrix} \bar{A}(h) & \bar{D} \\ I & 0 \\ 0 & I \end{bmatrix} < 0 \tag{32}$$

where

$$\Phi = \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -U(h) & P(h)\bar{E} \\ \bar{E}^TP(h) & \bar{\omega}_l^2\bar{E}^TU(h)\bar{E} \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & \bar{L} & 0 \\ 0 & 0 & I \end{bmatrix}^T \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} 0 & \bar{L} & 0 \\ 0 & 0 & I \end{bmatrix} \tag{33}$$

Define

$$\mathcal{X} = \begin{bmatrix} G^T & G^T & 0 \end{bmatrix}^T; \ \Lambda = \begin{bmatrix} -I & \bar{A} & \bar{D} \end{bmatrix}; \ \Gamma = I$$

Based on Lemma 1, then

$$\Sigma + \Gamma^T\mathcal{X}\Lambda + \Lambda^T\mathcal{X}^T\Gamma < 0 \tag{34}$$

As can see $\Gamma^\perp = 0$, the first inequality in condition (ii) of Lemma 1 disappears, and according to Lemma 1, (33) is valid for some $\mathcal{X}$ if and only if the second inequality of condition (ii) of lemma 1. Note that $\Lambda^\perp$ can be chosen as $\Lambda^\perp = \begin{bmatrix} \bar{A}(h) & \bar{D} \\ I & 0 \\ 0 & I \end{bmatrix}$ and by using the Schur complement, can get the equivalence between equations (29) and (30). Secondly, let's show that the (28) is equivalent to (31), Define (28) be rewritten in the form

$$\begin{bmatrix} \bar{A}(h) \\ I \end{bmatrix}^T \begin{bmatrix} 0 & Q(h)\bar{E} + SR \\ (Q(h)\bar{E} + SR)^T & 0 \end{bmatrix} \begin{bmatrix} \bar{A}(h) \\ I \end{bmatrix} < 0 \tag{35}$$

Let :

$$\Sigma = \begin{bmatrix} 0 & Q(h)\bar{E} + SR \\ (Q(h)\bar{E} + SR)^T & 0 \end{bmatrix}; \ G = \begin{bmatrix} G(h) \\ G(h) \end{bmatrix};$$
$$\mathcal{X} = \begin{bmatrix} -I & \bar{A}(h) \end{bmatrix}; \ \mathcal{X}^\perp = \begin{bmatrix} \bar{A}(h) \\ I \end{bmatrix} \tag{36}$$

So get that equations (35) and (36) are equivalent to inequality (31).

Theorem 2. For unknown actuator fault matrix F, closed-loop fuzzy system (24) is asymptotically stable, with H_∞ performance index γ in FF domain $|\omega| \le \bar{\omega}_l$, if there exist symmetric matrices $\tilde{U}_{1ti}$, $\tilde{U}_{2vi}$, $\tilde{U}_{33i}$, $\tilde{P}_{1ti}$, $\tilde{P}_{2vi}$, $\tilde{P}_{33i}$, $\tilde{Q}_{1ti}$, $\tilde{Q}_{2vi}$ and $\tilde{Q}_{33i}$ and matrices $\tilde{G}_{ti}$, $\tilde{S}_t$, Z_{si} and H, (with, $s = 1, 2, 3$, $t = 2, 3$, $v = 1, 2$) satisfying :

$$\Psi_{ii} < 0; \quad \Upsilon_{ii} < 0; \quad i = 1, ..., 4$$
$$\frac{1}{3}\Psi_{ii} + \frac{1}{2}\{\Psi_{ij} + \Psi_{ji}\} < 0; \quad 1 \le i \ne j \le 4 \tag{37}$$
$$\frac{1}{3}\Upsilon_{ii} + \frac{1}{2}\{\Upsilon_{ij} + \Upsilon_{ji}\} < 0; \quad 1 \le i \ne j \le 4$$

where

$$\Psi_{ij} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} \\ \star & \Psi_{22} & \Psi_{23} \\ \star & \star & \Psi_{33} \end{bmatrix}; \quad \Upsilon_{ij} = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ \star & \Upsilon_{22} \end{bmatrix};$$

$$\Psi_{11} = \begin{bmatrix} a_1 & -\bar{G}_{2i}^T - \tilde{U}_{12i} & -\bar{G}_{3i}^T - \tilde{U}_{13i} \\ \star & -H - H^T - \tilde{U}_{22i} & -H - \tilde{U}_{23i} \\ \star & \star & -H - H^T - \tilde{U}_{33i} \end{bmatrix};$$

$$\Psi_{12} = \begin{bmatrix} a_2 & a_3 & a_4 \\ a_5 & a_6 & -U - U^T \\ a_7 & a_8 & -L\bar{G}_{3i}^T + H^T - H \end{bmatrix}; \ \Psi_{13} = \begin{bmatrix} D & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$\Psi_{22} = \begin{bmatrix} a_9 & a_{10} & a_{11} \\ \star & \omega_l^2 \tilde{U}_{22i} - H - H^T & a_{12} \\ \star & \star & a_{13} \end{bmatrix}; \ \Psi_{23} = \begin{bmatrix} D & 0 & 0 \\ 0 & I & H^T \\ 0 & 0 & 0 \end{bmatrix};$$

$$\Psi_{33} = \begin{bmatrix} -\gamma^2 I & 0 & 0 \\ 0 & -\gamma^2 I & 0 \\ 0 & \star & -I \end{bmatrix};$$

$$\Upsilon_{11} = \begin{bmatrix} -\bar{G}_{1i} - \bar{G}_{1i}^T & -\bar{G}_{2i}^T & -\bar{G}_{3i}^T - DH \\ \star & -H - H^T & -H \\ \star & \star & -H - H^T \end{bmatrix};$$

$$\Upsilon_{12} = \begin{bmatrix} a_{14} & a_{15} & a_{16} \\ a_{17} & \tilde{Q}_{22i} + \tilde{S}_2^T R_2^T - H - H^T & a_{18} \\ a_{19} & \tilde{S}_3^T R_2^T - H - H^T & a_{20} \end{bmatrix};$$

$$\Upsilon_{22} = \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ \star & -H - H^T & -H^T - \bar{G}_{2i} L^T + H \\ \star & \star & a_{24} \end{bmatrix};$$

$$R_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ I & I \end{bmatrix}; \ R_2 = R_3 = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}; V = \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix};$$

$$a_1 = -\tilde{U}_{11i} - \bar{G}_{1i} - \bar{G}_{1i}^T;$$
$$a_2 = \tilde{P}_{11i} + A_j \bar{G}_{1i}^T + \delta_1 B Z_{1i} V^T + \Omega \delta_2 B Z_{1i} V^T - \bar{G}_{1i};$$
$$a_3 = \tilde{P}_{12i} + A_j \bar{G}_{2i}^T + \delta_1 B Z_{1i} + \Omega \delta_2 B Z_{1i} + \delta_1 B Z_{2i} + \Omega \delta_2 B Z_{2i};$$
$$a_4 = A_j \bar{G}_{3i}^T + \delta_1 B Z_{1i} + \Omega \delta_2 B Z_{1i} + \delta_1 B Z_{2i} + \Omega \delta_2 B Z_{2i} - VH;$$
$$a_5 = \tilde{P}_{12i}^T - \bar{G}_{2i} - H^T V^T; \ a_6 = -H - H^T + \tilde{P}_{22i}^T;$$
$$a_7 = \tilde{P}_{13i}^T - L\bar{G}_{1i}^T - \bar{G}_{3i}; \ a_8 = \tilde{P}_{23i}^T - L\bar{G}_{2i}^T + H^T;$$
$$a_9 = \omega_l^2 \tilde{U}_{11i} + sym[A_j \bar{G}_{1i}^T + \delta_1 B Z_{1i} V^T + \Omega \delta_2 B Z_{1i} V^T];$$
$$a_{10} = \omega_l^2 \tilde{U}_{12i} + A_j \bar{G}_{2i}^T + \delta_1 B Z_{1i} + \Omega \delta_2 B Z_{1i} + \delta_1 B Z_{2i} + \Omega \delta_2 B Z_{2i} - VH;$$
$$a_{11} = A_j \bar{G}_{3i}^T + \delta_1 B Z_{2i} + \Omega \delta_2 B Z_{2i} - \bar{G}_{1i} L^T + VH;$$
$$a_{12} = -\bar{G}_{2i} L^T + H - H^T; \ a_{13} = -\bar{G}_{3i} L^T - L\bar{G}_{3i}^T + H^T + H;$$
$$a_{14} = \tilde{Q}_{11i} + \tilde{S}_1^T R_1^T + \bar{G}_{1i} A_j^T + V Z_{1i}^T B^T \delta_1 + V Z_{1i}^T B^T \Omega \delta_2 - \bar{G}_{1i}^T;$$
$$a_{15} = \tilde{Q}_{12i} + \tilde{S}_1^T R_2^T - VH - \bar{G}_{2i}^T;$$
$$a_{16} = \tilde{Q}_{13i} + \tilde{S}_1^T R_3^T - \bar{G}_{1i} L^T + VH - \bar{G}_{3i}^T;$$
$$a_{17} = \tilde{Q}_{12i}^T + \tilde{S}_2^T R_1^T + \bar{G}_{2i} A_j^T + Z_{1i}^T B^T \delta_1 + Z_{1i}^T B^T \Omega \delta_2 + Z_{2i}^T B^T \delta_1 + Z_{2i}^T B^T \Omega \delta_2;$$
$$a_{18} = \tilde{Q}_{23i} + \tilde{S}_2^T R_3^T - \bar{G}_{2i} L^T + H;$$
$$a_{19} = \tilde{S}_3^T R_1^T + \bar{G}_{3i} A_j^T + Z_{2i}^T B^T \delta_1 + Z_{2i}^T B^T \Omega \delta_2 - H^T V^T;$$
$$a_{20} = \tilde{S}_3^T R_3^T - \bar{G}_{3i} L^T + H - H^T;$$
$$a_{21} = sym[A_j \bar{G}_{1i}^T + \delta_1 B Z_{1i} V^T + \Omega \delta_2 B Z_{1i} V^T];$$
$$a_{22} = A_j \bar{G}_{2i}^T + \delta_1 B Z_{1i} + \Omega \delta_2 B Z_{1i} + \delta_1 B Z_{2i} + \Omega \delta_2 B Z_{2i} - VH;$$
$$a_{23} = A_j \bar{G}_{3i}^T + \delta_1 B Z_{2i} + \Omega \delta_2 B Z_{2i} - \bar{F}_{1i} L^T + VH$$
$$a_{24} = -\bar{G}_{3i} L^T - L\bar{G}_{3i}^T + H^T + H;$$

Furthermore, under the above conditions, the fuzzy H_∞ SOF controller gain matrices in (22) are

$$K_{1i} = Z_{1i} H^{-1}; \quad K_{2i} = Z_{2i} H^{-1}; \ 1 \le i \le 4 \qquad (38)$$

Proof 2. Left- and right- multiplying (30) and (31) by $diag\{G^{-1}(h),\ G^{-1}(h),\ I,\ I\}$ and $diag\{G^{-1}(h),\ G^{-1}(h)\}$, respectively.
Let $\bar{G}(h) = G^{-1}(h)$, $\tilde{U}(h) = G^{-1}(h)U((h)G^{-T}(h)$, $\tilde{P}(h) = G^{-1}(h)P(h)G^{-T}(h)$, $\tilde{Q}(h) = G^{-1}(h)Q(h)G^{-T}(h)$ and $\tilde{S} = G^{-1}(h)SG^{-T}(h)$.
Furthermore, Defining the following matrices :

$$\tilde{U}(h) = \sum_{i=1}^{4} h_i \begin{pmatrix} \tilde{U}_{11i} & \tilde{U}_{12i} & \tilde{U}_{13i} \\ \star & \tilde{U}_{22i} & \tilde{U}_{23i} \\ \star & \star & \tilde{U}_{33i} \end{pmatrix};$$

$$\tilde{P}(h) = \sum_{i=1}^{4} h_i \begin{pmatrix} \tilde{P}_{11i} & \tilde{P}_{12i} & \tilde{P}_{13i} \\ \star & \tilde{P}_{22i} & \tilde{P}_{23i} \\ \star & \star & \tilde{R}_{33i} \end{pmatrix};$$

$$\tilde{Q}(h) = \sum_{i=1}^{4} h_i \begin{pmatrix} \tilde{Q}_{11i} & \tilde{Q}_{12i} & \tilde{Q}_{13i} \\ \star & \tilde{Q}_{22i} & \tilde{Q}_{23i} \\ \star & \star & \tilde{Q}_{33i} \end{pmatrix};$$

$$\bar{G}(h) = \sum_{i=1}^{4} h_i \begin{pmatrix} \bar{G}_{1i} & 0 & VH \\ \bar{G}_{2i} & H & H \\ \bar{G}_{3i} & 0 & H \end{pmatrix}; \ \tilde{S} = \begin{pmatrix} \tilde{S}_1^T \\ \tilde{S}_2^T \\ \tilde{S}_3^T \end{pmatrix}^T;$$

$$\bar{A}(h) = \sum_{i=1}^{4} \sum_{j=1}^{4} h_i h_j \begin{pmatrix} A_j & FB_j K_{1i} & FB_j K_{2i} \\ 0 & 0 & -I \\ -L & 0 & I \end{pmatrix}$$

After that we applying Lemma 2, we have (30) and (31) are equivalent to (37).

Table 1: Numerical values of Diesel engine

T_i	Intake manifold temperature	340 K
T_a	Ambient temperature	310 K
p_a	Ambient pressure	100 kPa
T_x	Exhaust manifold temperature	620 K
N	Engine Speed	1500 rpm
V_i	Volume of the intake manifold	0.003 m^3
V_d	Displacement volume	0.006 m^3
c_p	Specific heat at constant pressure	1014,4 $J/Kg.K$
c_v	Specific heat at volume pressure	727,4 $J/Kg.K$
η_c	compressor efficiency	0.61
V_x	Volume of the exhaust manifold	0.002 m^3
τ	Time constant	0.11
η_t	turbine efficiency	0.76
η_v	Volumetric efficiency	0.87
R	Gas constant	287 $J/Kg.K$
η_m	Turbocharger mechanical efficiency	1

4. SIMULATION

In this section, results are presented to illustrate the effectiveness of the proposed controller design. The Diesel engine air-path model parameters are given in Table 1. The actuator fault is taken as $0.25 \le F_1 \le 0.75$, $0.35 \le F_2 \le 0.65$. Choosing $p_i \in [135kPa, 160kPa]$ and $p_x \in [85kPa, 115kPa]$ and fuel flow rate are given as follows :

$$m_f = \begin{cases} 6/3600 & 10 \le t \le 20 \\ 3/3600 & 20 \le t \le 30 \\ 4/3600 & others \end{cases} \qquad (39)$$

Since the frequency range of the disturbances can be seen as belonging to the FF domain $|\omega| \le 0.5$ rad/s.
We set the initial conditions to $p_i(0) = 145\ kPa$, $p_x(0) =$

90 kPa and $p_c(0) = 1200\ W$. The frequency responses of the superposition of the measurements of the real outputs system P_i, P_x and its reference are plotted in figs. 3 and 4, respectively, the solid line being obtained by the FF FTSOFC method, while the dotted line is obtained by the EF FTSOFC approach. Can conclude from these two figures that the frequency response results validate the H_∞ performance, good tracking and fast convergence of the proposed FF FTSOFC method in this article.

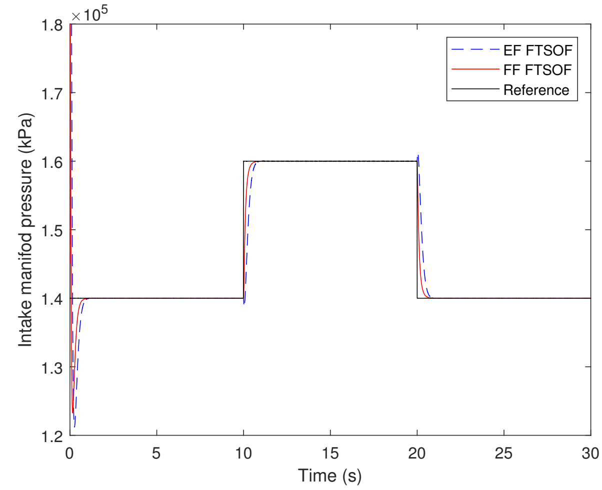

Fig. 3. Intake manifold pressures

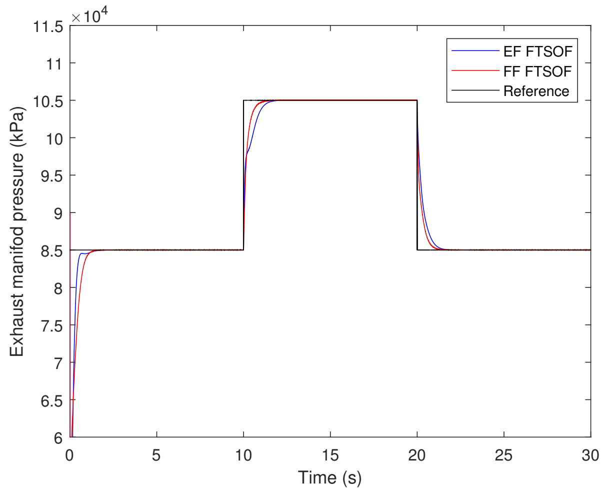

Fig. 4. Exhaust manifold pressures

5. CONCLUSION

In this work, a fault- tolerant finite-frequency H_∞ static output control is proposed for the Diesel engine air-path system. The LMI sufficient conditions guaranteeing good tracking and specified H_∞ performance are obtained with actuator faults. The Air-path Diesel engine model is described by T-S fuzzy model based descriptor approach. Simulation has been provided to illustrate the effectiveness of the proposed approach.

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REFERENCES

Chadli, M., Shi, P., Feng, Z., & Lam, J. (2017). New bounded real lemma formulation and control for continuous-time descriptor systems. *Asian Journal of Control*, 19(6), 2192-2198.

Cheng, L., Dimitriou, P., Wang, W., Peng, J., & Aitouche, A. (2020). A novel fuzzy logic variable geometry turbocharger and exhaust gas recirculation control scheme for optimizing the performance and emissions of a diesel engine. *International Journal of Engine Research*, 21(8): 1298-1313.

El-Amrani, A., El Hajjaji, A., & Aitouche, A. (2022, November). Finite Frequency Static Output Feedback H_∞ Control Design of a Urea-Selective Catalytic Reduction Aftertreatment system. *In 2022 10th International Conference on Systems and Control (ICSC)*, 289-293, IEEE.

El-Amrani, A., El Hajjaji, A., & Bosche, J. (2023). Finite Frequency Fuzzy Static Output Feedback H_∞ Control for Diesel Engine Air-Path: Descriptor Approach. *In 2023 IEEE International Conference on Fuzzy Systems (FUZZ)*, 1-7.

Elias, L. J., Faria, F. A., Araujo, R., Magossi, R. F., & Oliveira, V. A. (2022). Robust Static Output Feedback H_∞ Control for Uncertain Takagi–Sugeno Fuzzy Systems. *IEEE Transactions on Fuzzy Systems*, 30(10): 4434-4446.

Iwasaki, T., & Hara, S. (2005). Generalized KYP lemma: Unified frequency domain inequalities with design applications. *IEEE Transactions on Automatic Control*, 50(1), 41-59.

Gahinet, P., & Apkarian, P. (1994). A linear matrix inequality approach H_∞ control. *IEEE Transactions on Systems, Man, and Cybernetics*, 4(4) : 421–44.

Stefanopoulou, A. G., Kolmanovsky, I., & Freudenberg, J. S. (2000). Control of variable geometry turbocharged diesel engines for reduced emissions. *IEEE transactions on control systems technology*, 8(4), 733-745.

Selvaraj, P., Kwon, O. M., Lee, S. H., & Sakthivel, R. (2023). Robust fault-tolerant control design for polynomial fuzzy systems. *Fuzzy Sets and Systems*, 464: 108406.

Takagi, T., & Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control. *IEEE Transactions on Systems, Man, and Cybernetics*, 15(1) : 116-132.

Tuan, H. D., Apkarian, P., Narikiyo, T., & Yamamoto, Y. (2001). Parameterized linear matrix inequality techniques in fuzzy control system design. *IEEE Transactions on fuzzy systems*, 9(2),324-332.

Tu, W., & Dong, J. (2023). Improved robust reduced-order sliding mode fault-tolerant control for TS fuzzy systems with disturbances. *Fuzzy Sets and Systems*, 464:108481.

Yang, G. H., Wang, J. L., & Soh, Y. C. (2001). Reliable H_∞ controller design for linear systems. *Automatica*, 37(5), 717-725.